Electromagnetic amplification by means of noble metal surface plasmon polaritons excited by electron beam

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Abstract: In this paper, the electromagnetic wave amplification in the frequency regime from visible light to ultra-violet based on noble metal surface plasmon polaritons (SPPs) excited by electron beam is presented. The theoretical analyses and numerical calculations show that SPPs can be amplified by the interaction between the electron beam and the waves, provided the beam velocity is a little bit larger than the phase velocity of SPPs wave. The gain of the amplification reaches up to about 10 *dB/cm* for the current density 200 A/cm^2 or even lower. When the beam velocity is larger than the Cherenkov radiation threshold, the SPPs can be transformed into radiation. The results of numerical calculations also show that promising amplification gain can also be obtained in the structure with parameters larger than nanometers, and this is of great significance for practical manufacture and applications.

Keywords: Amplification, Surface plasmon polaritons, Electron beam

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I. Introduction

The amplification of micro-waves and millimeter waves based on the interaction between the waves and electron beams are often used in traditional vacuum electronic devices, such as the traveling-wave tube, klystron [1-6], etc. However, for terahertz or even higher frequency, it becomes very difficult because the size of the slow wave structures should be very small, and the start current of oscillation should be very high. In this paper, a novel mechanism for light wave amplification is presented based on SPPs and free electron beams. It is similar to theory of relativistic space charge waves [7-9].

SPPs are electromagnetic waves highly confined on the metal dielectric interface [10, 11]. The promising properties of SPPs have led to various applications in modern science and technology, such as biological and chemical sensing [12-14], near field imaging [15, 16], and enhanced optical transmission [17], etc. As the nature link between optics and electronics, SPPs are recently used to combine the two subjects to overcome the terahertz gap [18-22], which is still a great challenge for electromagnetic science.

In this paper, SPPs is used as the slow waves to realize the amplification by the interaction between the waves and beams. The theoretical analyses and numerical calculations given in Section II and III show that waves can be effectively amplified based on SPPs and electron beams in the frequency regime from visible light to ultra-violet, if the beam velocity is slightly larger than the phase velocity of SPPs, just similar to the relativistic space charge theory [7-9]. This is of great significance for electronic science and applications of SPPs.

II. Analytical theory

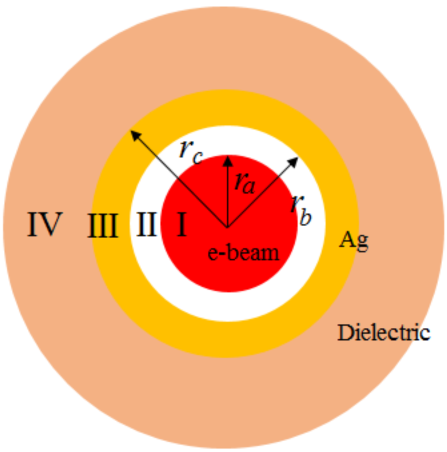

Fig. 1 The schematic of the structure

The schematic of the structure is shown in figure.1. The electron beam is moving inside the hollow metal film, and the loading dielectric is outside. In this paper, Ag is used, and its permittivity is described by the modified Drude model [23]:

$$\varepsilon_{Ag} = \varepsilon_{\infty} - \frac{\omega_p^{\ 2}}{\omega^2 + j\omega\gamma} \tag{1}$$

where $\varepsilon_{\infty} = 5.3$, $\omega_p = 1.39e16\ rad/s$, and $\gamma = 3.21e13\ Hz$. In addition, we also consider that the temperature of the structure is not changed in the whole physical process. To analyses the amplification based on SPPs of the metal, the structure can be divided into four regions as shown in figure.1. Based on the Maxwell equations and the relativistic space charge wave theory [7-9], the field equations in each region can be found.

Region I is the electron beam:

$$\begin{cases} E_z^{\ I} = A_a^{\ 1} I_0\left(k_1 r\right) e^{jk_z z} e^{-j\omega t} \\ H_\theta^{\ I} = -\dfrac{j\omega\varepsilon_1\varepsilon_0}{k_1} A_a^{\ 1} I_1\left(k_1 r\right) e^{jk_z z} e^{-j\omega t} \end{cases} \tag{2}$$

where $k_1 = \sqrt{\left(k_z^{\ 2} - \varepsilon_1 k_0^{\ 2}\right)\alpha}$, $\alpha = 1 - \omega_{pe}^{\ 2} / \left(\omega - k_z u_0\right)^2$, $\omega_{pe}^{\ 2} = \left(e\rho_0\right)/\left(m_e\varepsilon_0\right)$, u_0 and ρ_0 are the velocity and the charge density of the electron beam, respectively. $I_0\left(x\right)$ and $I_1\left(x\right)$ are the modified Bessel functions.

Region II is the vacuum between the electron beam and the Ag film:

$$\begin{cases} E_z^{\ II} = \left(A_a^{\ II} I_0\left(k_2 r\right) + A_b^{\ II} K_0\left(k_2 r\right)\right) e^{jk_z z} e^{-j\omega t} \\ H_\theta^{\ II} = -\dfrac{j\omega\varepsilon_2\varepsilon_0}{k_2} \left(A_a^{\ II} I_1\left(k_2 r\right) - A_b^{\ II} K_1\left(k_2 r\right)\right) e^{jk_z z} e^{-j\omega t} \end{cases} \tag{3}$$

where $k_2 = \sqrt{k_z^{\ 2} - \varepsilon_2 k_0^{\ 2}}$, $K_0\left(x\right)$ and $K_1\left(x\right)$ are the modified Bessel functions.

region III is the Ag film:

$$\begin{cases} E_z^{\ III} = \left(A_a^{\ III} I_0\left(k_3 r\right) + A_b^{\ III} K_0\left(k_3 r\right)\right) e^{jk_z z} e^{-j\omega t} \\ H_\theta^{\ III} = -\dfrac{j\omega\varepsilon_{Ag}\varepsilon_0}{k_3} \left(A_a^{\ III} I_1\left(k_3 r\right) - A_b^{\ III} K_1\left(k_3 r\right)\right) e^{jk_z z} e^{-j\omega t} \end{cases} \tag{4}$$

where $k_3 = \sqrt{k_z^{\ 2} - \varepsilon_{Ag} k_0^{\ 2}}$.

Region IV is the dielectric loading outside of the Ag hollow film:

$$\begin{cases} E_z^{IV} = A_b^{IV} K_0\left(k_4 r\right) e^{jk_z z} e^{-j\omega t} \\ H_\theta^{IV} = \dfrac{j\omega\varepsilon_4\varepsilon_0}{k_4} A_b^{IV} K_1\left(k_4 r\right) e^{jk_z z} e^{-j\omega t} \end{cases} \tag{5}$$

where $k_4 = \sqrt{{k_z}^2 - \varepsilon_4 {k_0}^2}$.

Based on the boundary conditions, the dispersion equation of the structure considering electron beam can be found as:

$$\frac{\varsigma_a I_0\left(k_2 r_b\right) + \varsigma_b K_0\left(k_2 r_b\right)}{\varsigma_a \varepsilon_2 k_3 I_1\left(k_2 r_b\right) - \varsigma_b \varepsilon_2 k_3 K_1\left(k_2 r_b\right)} = \frac{\xi_a I_0\left(k_3 r_b\right) + \xi_b K_0\left(k_3 r_b\right)}{\xi_a \varepsilon_{Ag} k_2 I_1\left(k_3 r_b\right) - \xi_b \varepsilon_{Ag} k_2 K_1\left(k_3 r_b\right)} \tag{6}$$

where

$$\varsigma_a = \frac{\varepsilon_2 k_1 I_0\left(k_1 r_a\right) K_1\left(k_2 r_a\right) + \varepsilon_1 k_2 I_1\left(k_1 r_a\right) K_0\left(k_2 r_a\right)}{\varepsilon_2 k_1 I_0\left(k_2 r_a\right) K_1\left(k_2 r_a\right) + \varepsilon_2 k_1 I_1\left(k_2 r_a\right) K_0\left(k_2 r_a\right)}$$

$$\varsigma_b = \frac{\varepsilon_2 k_1 I_0\left(k_1 r_a\right) I_1\left(k_2 r_a\right) - \varepsilon_1 k_2 I_0\left(k_2 r_a\right) I_1\left(k_1 r_a\right)}{\varepsilon_2 k_1 I_1\left(k_2 r_a\right) K_0\left(k_2 r_a\right) + \varepsilon_2 k_1 I_0\left(k_2 r_a\right) K_1\left(k_2 r_a\right)}$$

$$\xi_a = \frac{\varepsilon_{Ag} k_4 K_0\left(k_4 r_c\right) K_1\left(k_3 r_c\right) - \varepsilon_4 k_3 K_0\left(k_3 r_c\right) K_1\left(k_4 r_c\right)}{\varepsilon_{Ag} k_4 I_0\left(k_3 r_c\right) K_1\left(k_3 r_c\right) + \varepsilon_{Ag} k_4 I_1\left(k_3 r_c\right) K_0\left(k_3 r_c\right)}$$

$$\xi_b = \frac{\varepsilon_{Ag} k_4 I_1\left(k_3 r_c\right) K_0\left(k_4 r_c\right) + \varepsilon_4 k_3 I_0\left(k_3 r_c\right) K_1\left(k_4 r_c\right)}{\varepsilon_{Ag} k_4 I_1\left(k_3 r_c\right) K_0\left(k_3 r_c\right) + \varepsilon_{Ag} k_4 I_0\left(k_3 r_c\right) K_1\left(k_3 r_c\right)}$$

Considering the energy loss of the metal and the interaction of the SPPs and electron beam, k_z in these equations can be written as:

$$k_z = k_z^{\,r} + jk_z^{\,i} \tag{7}$$

When $k_z^{\,i}$ is positive, the wave vector reads $e^{jk_z^{\,r} z - k_z^{\,i} z} e^{-j\omega t}$, which indicates that the SPPs propagate along the film with attenuation. And when $k_z^{\,i}$ is negative, the wave vector reads $e^{jk_z^{\,r} z + k_z^{\,i} z} e^{-j\omega t}$, which indicates that the SPPs propagate with amplification. The gain of the amplification can be obtained by the imaginary part of kz.

$$G = 20\log_{10}(e^{\mathrm{Im}(k_z)L}) \tag{8}$$

III. Numerical calculation

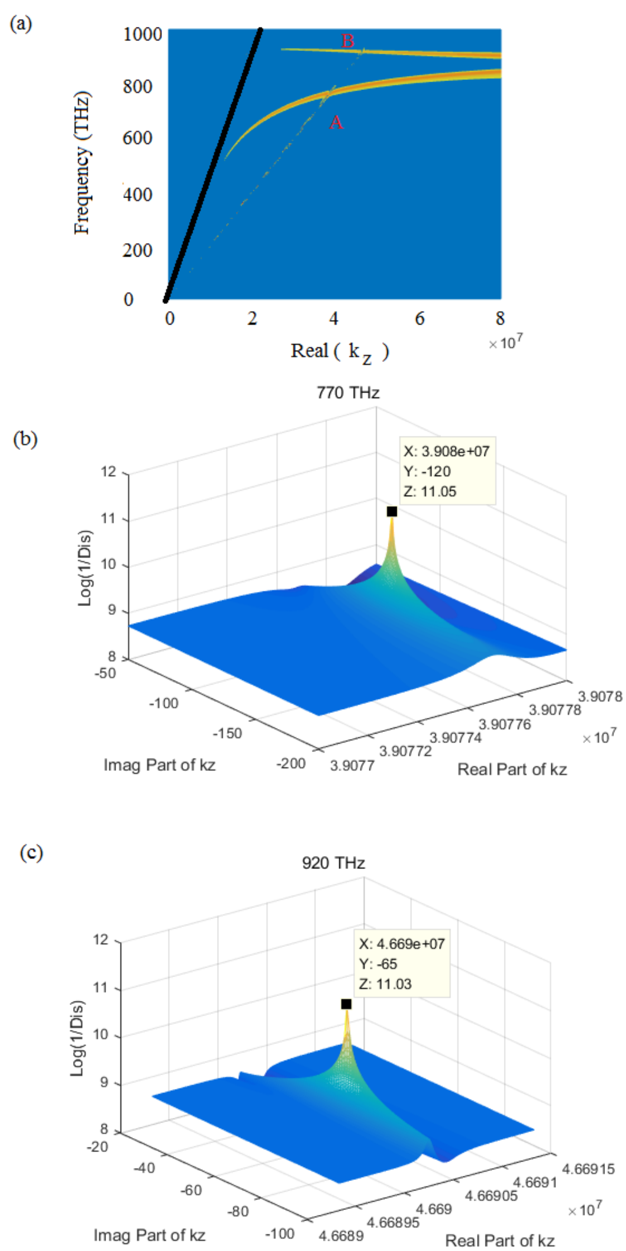

Fig. 2 (a) The dispersion curve of the structure; (b) The value of k_z at the working point A (770 *THz*); (c) The value of k_z at the working point B (920 *THz*).

The dispersion curve of structure is shown in figure.2 (a), in which the radius of the electron beam is 100 *nm*. The inner and the outer radius of the hollow Ag film are 120 *nm* and 140 *nm* respectively. The permittivity of the dielectric is 1. The energy and the current density of the electron beam are 50 *keV* and 200 A/cm^2. It can be seen that there are two working points corresponding to the two modes of the SPPs of the Ag film. When the velocity of the electron beam is a little larger than the phase velocity of the SPPs, the energy of the electron beam can be transferred to the SPPs by the interaction between the waves and the beam. The value of the k_z at the working points A and B are shown in figure.2 (b) and (c), and they are 3.9e7-1j*120 and 4.67e7-1j*65, respectively. The negative imaginary part of k_z indicates that the SPPs propagate along Z direction with amplification.

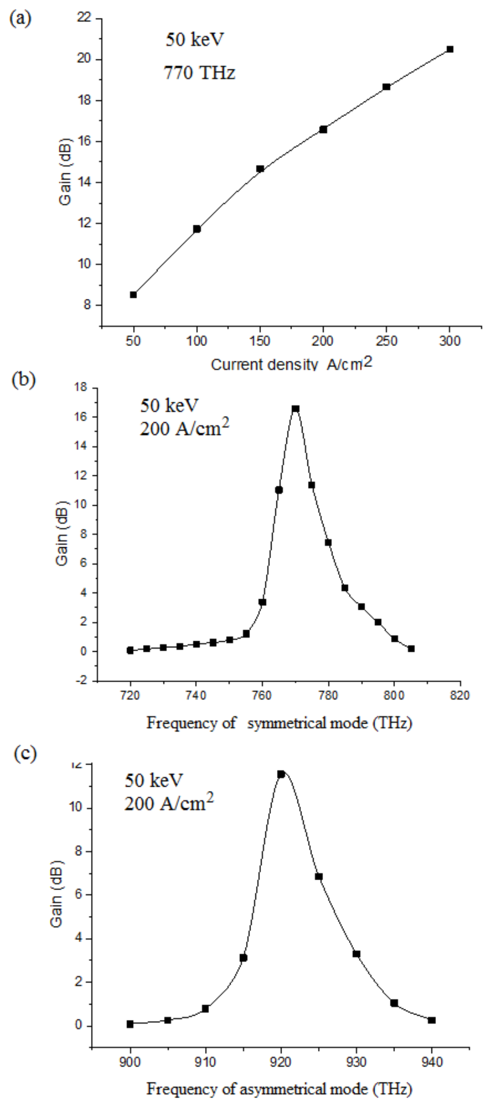

Fig. 3 (a) Gain for different current density; (b) Gain for different frequency; (c) Gain for different frequency.

Figure. 3 (a) shows the gain of the SPPs amplification for different electron beam current density with the same beam energy 50 *keV* and operating frequency 770 *THz*.

It can be seen that the gain increases with the increase of the current density, and it reaches 17 *dB/cm* and 9 *dB/cm* for 200 A/cm^2 and 50 A/cm^2, respectively. This is because the larger current density leads stronger interaction between the beam and the SPPs waves. The gain at different frequency components of the two SPPs modes are shown in figure. 3(b) and (c). The SPPs, whose phase velocity are a little larger than the electron beam, get the largest gain. It can be seen that the gain of the symmetrical mode (XX THz) reaches XX *dB/cm* and asymmetrical mode (XX THz) XX *dB/cm*, and the 3 *dB* band of the gain reach 20 *THz* and 17 *THz* for the two SPPs modes, respectively.

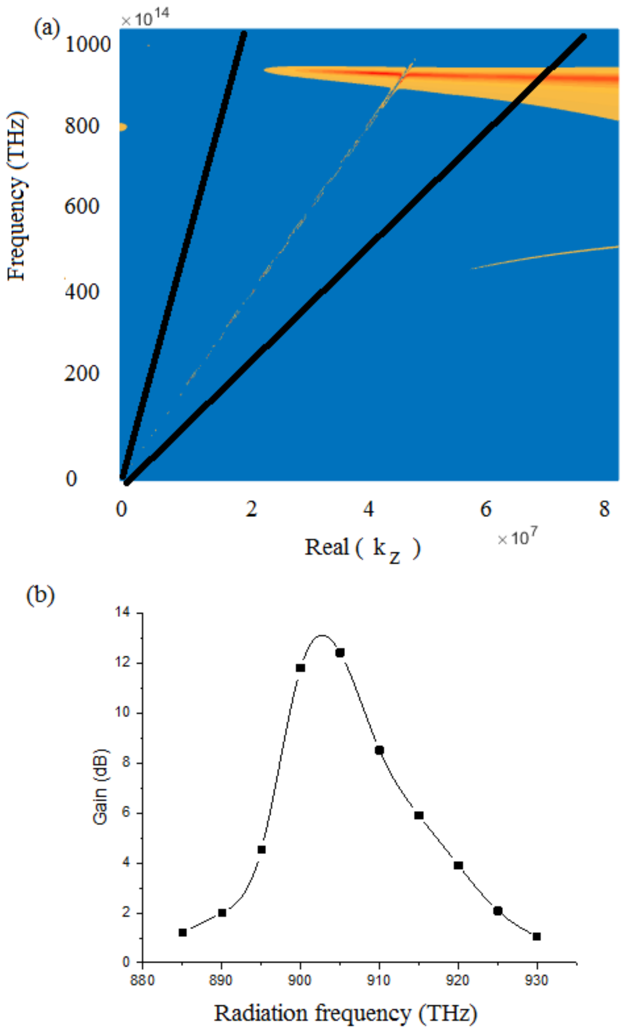

Fig. 4 (a) The dispersion curves, and the permittivity of the loading dielectric is 9; (b) The gain for different frequency

Figure.4 (a) shows the dispersion curve when the permittivity of the loading dielectric is 9 for electron beam energy 50 *keV* and current density 200 A/cm^2.The imaginary part of kz is negative, which means that the SPPs on the metal film not only propagate but also be amplified along the metal film. Study shows that when the electron beam energy is larger than the Cherenkov radiation threshold, the SPPs can be transformed into coherent and tunable Cherenkov radiation. For beam energy 50 *keV*, the working point is located in the radiation region (the region between the light line and the dielectric line), this indicates the amplified SPPs can also be transformed to get the coherent, tunable and amplified Cherenkov radiation. Further numerical calculations show that the gain in this case reach up to 13 *dB/cm*, and the 3 *dB* gain band reaches to 20 *THz*, as shown in figure.4 (b).

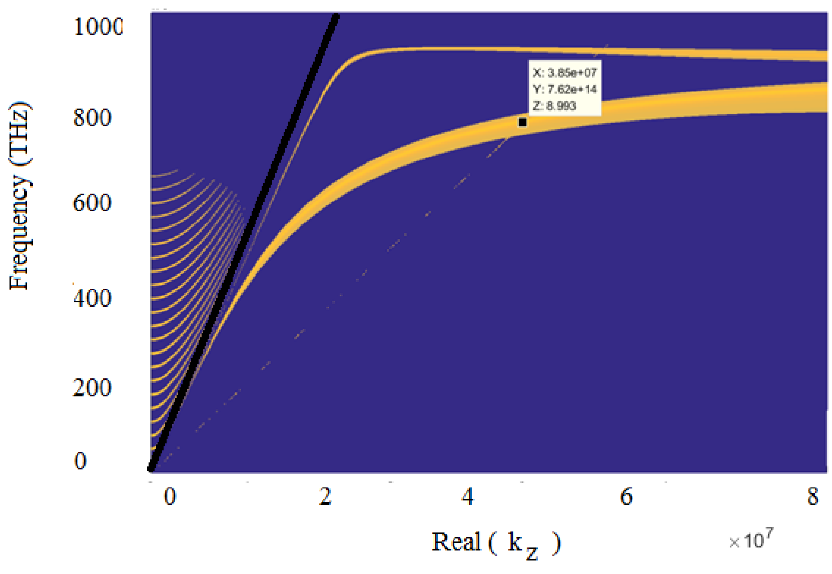

Fig. 5 The dispersion curve for inner radius 5μm.

From the viewpoint of practical manufacture and applications, the structure with much larger size are also studied in this paper. The dispersion curve and the kz value for the structure with inner radius 5 μm are shown in figure 5. The numerical calculations show that the SPPs can also be amplified with these structure parameters, and the kz of the SPPs 762 *THz* is kz=3.882e7-1j*89, corresponding to the gain 7.8 *dB/cm*.

IV. Conclusion

In this paper, the amplification in the frequency regime from visible light to ultra-violet based on the interaction between Ag film SPPs and electron beam is presented. The theoretical analyses and numerical calculations show that SPPs can be amplified by the interaction between the electron beam and the waves. The gain of the amplification reaches up to about 10 *dB/cm* with a 3 *dB* gain band about tens of terahertz for the current density 200 A/cm^2 or even lower. When the electron beam velocity is larger than the Cherenkov radiation threshold, the SPPs can be not only transformed into radiation but also amplified by the electron beam. The theoretical study shows that when the velocity of the electron beam is a little larger than the phase velocity of SPPs, the surface plasmon wave is amplified. This is quite similar to the relativistic space charge wave [7-9]. And the amplification gain for the radiation waves also reaches up to 13 *dB/cm* for current density 200 A/cm^2. The results of numerical calculations also show that promising amplification gain can also be obtained in the structure with parameters much larger than nanometers. This is of great significance for practical manufacture and applications. Comparing to the papers on the amplifiers in terahertz or even higher frequency regime, appeared in reference [24-28], the amplifier based on our novel mechanism are much simpler and more practical.

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