Terahertz amplification by electron beam excited graphene surface plasmon polaritons in a cylindrical structure

Zhao Tao[1, 2*], Gong Sen[1, 2], Hu Min [1, 2], Zhong Renbin [1, 2], Zhang Ping [1, 2], Yu Chengpeng [1, 2], and Liu Shenggang [1, 2]

[1] Terahertz Science and Technology Research Center, University of Electronic Science and Technology of China, Chengdu, Sichuan, China, 610054

[2] Cooperative Innovation Centre of Terahertz Science, National 2011 project

[*] Email: forzhaotao@126.com

(Received September 10, 2016)

Abstract: Graphene surface plasmon polaritons (SPPs) exhibit strong mode confinement and long propagation distances, and the characteristics are tunable via changing the chemical potential, etc. In this paper, we propose a novel physical mechanism that the graphene SPPs excited by electron beam can be used to amplify terahertz (THz) waves. Theoretical results show that the energy transfers from electron beam to the SPPs occurs when the velocity of electron beam is moderately faster than the phase velocity of SPPs. This is just the similar results by the space charge wave theory. The physical mechanism may provide great potential for graphene SPPs-based amplifiers.

Keywords: Electron beam, Surface plasmon polaritons, Amplifier, Terahertz, Cylindrical structure

doi: 10.11906/TST.101-108.2016.09.10

I. Introduction

The traveling wave can be amplified by electron beam when the velocity of the beam is moderately faster than the phase velocity of the wave. This physical phenomenon has been used for electromagnetic wave amplifications successfully for a long time in microwave and millimeter wave [1-6]. However, till now proper amplifiers in THz and higher frequencies regime are still remain a big challenge, even the graphene Surface Plasmon Polaritons (SPPs) have been used for THz radiation successfully [7-10]. SPPs are electromagnetic surface waves propagating along the metal-dielectric or graphene-dielectric interface. In this paper, we propose a novel physical mechanism that the graphene SPPs can be amplified by a moving electron beam. The energy transfer from electron beam to the SPPs can occur when the velocity of electron beam is moderately faster than the phase velocity of SPPs, and it is similar to the theory of space charge wave [1-6]. Therefore, THz amplifiers can be realized in THz and higher frequency regime.

II. Analytical theory

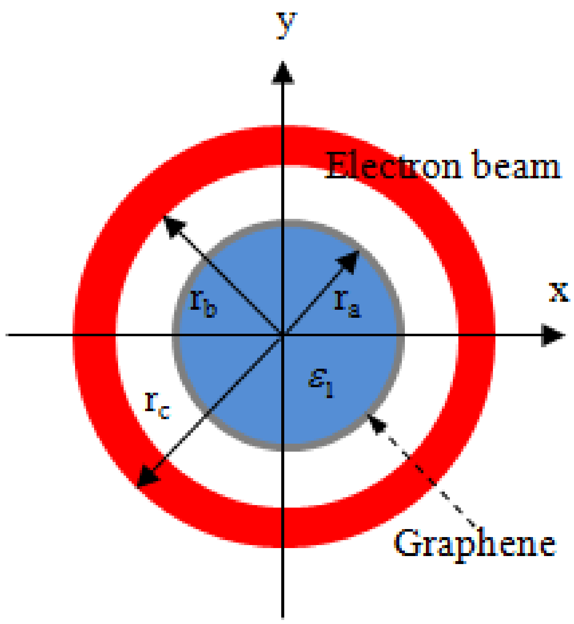

Fig. 1 Schematic of graphene SPPs amplifier. The graphene is coated on a dielectric rod, and the circular electron beam is moving with velocity v_0 outside the graphene-dielectric structure.

The structure of graphene SPPs amplifier is shown in Figure 1, and the moving circular electron beam can interact with the SPPs and transfer energy to the SPPs in the graphene-dielectric structure [10]. Solving the homogeneous Helmholtz equation together with the boundary conditions in the cylindrical coordinate system, the E_z component of the electromagnetic fields in region I and III can be obtained. And then all the other field components can be obtained by Maxwell's equations. The wave factor $e^{-j\omega t+jk_z z}$ is neglected.

In the region I:

$$\begin{aligned} E_z^I &= A_1 k_{c1}^2 I_0(k_{c1} r) \\ H_\varphi^I &= -A_1 j\omega\varepsilon_0\varepsilon_1 k_{c1} I_1(k_{c1} r) \end{aligned} \qquad (1)$$

Where：

$$k_{c1}=\sqrt{k_z^2-\varepsilon_1 k_0^2}$$

In the region II:

$$\begin{aligned} E_z^{II} &= A_3 k_{c2}^2 I_0(k_{c2}r)+A_4 k_{c2}^2 K_0(k_{c2}r) \\ H_\varphi^{II} &= -j\omega\varepsilon_0 k_{c2}[A_3 I_1(k_{c2}r)-A_4 K_1(k_{c2}r)] \end{aligned} \quad (2)$$

Where $k_{c2}=\sqrt{\alpha(k_z^2-k_0^2)},\ \alpha=1-\frac{\omega_{pe}^2}{(\omega-k_z v_0)^2} \quad \omega_{pe}^2=\frac{\rho_0 e}{\varepsilon_0 m},\ \rho_0=\frac{J}{v_0}$

In the region III:

$$\begin{aligned} E_z^{III} &= A_5 k_{c3}^2 I_0(k_{c3}r)+A_6 k_{c3}^2 K_0(k_{c3}r) \\ H_\varphi^{III} &= -j\omega\varepsilon_0 k_{c3}[A_5 I_1(k_{c3}r)-A_6 K_1(k_{c3}r)] \end{aligned} \quad (3)$$

Where： $k_{c3}=\sqrt{k_z^2-k_0^2}$

In the region IV:

$$\begin{aligned} E_z^{IV} &= A_7 k_{c4}^2 K_0(k_{c4}r) \\ H_\varphi^{IV} &= j\omega\varepsilon_0 k_{c4} A_7 K_1(k_{c4}r) \end{aligned} \quad (4)$$

Where： $k_{c4}=\sqrt{k_z^2-k_0^2}$

The monolayer graphene is also considered as a conductive surface with conductivity σ_g [11, 12],

$$\sigma_g=\frac{je^2 k_B T}{\pi\hbar^2(\omega+j/\tau)}\{\frac{\mu_c}{k_B T}+2\ln[\exp(-\mu_c/k_B T)+1]\} \quad (5)$$

j is the imaginary unit , T is temperature, k_B is Boltzmann constant, τ is relaxation time, and μ_c is chemical potential.

The boundary conditions can be written as

$$
\begin{aligned}
&E_z^{I} = E_z^{II}, H_\varphi^{II} - H_\varphi^{I} = \sigma_g E_z \\
&E_z^{II} = E_z^{III}, H_\varphi^{II} = H_\varphi^{III} \\
&E_z^{IV} = E_z^{III},\ H_\varphi^{III} = H_\varphi^{IV}
\end{aligned} \qquad (6)
$$

Submitting the electromagnetic fields into the above boundary conditions, the dispersion relation can be obtained:

$$\frac{I_1(k_{c3}r_c) - M_3 K_1(k_{c3}r_c)}{I_0(k_{c3}r_c) + M_3 K_0(k_{c3}r_c)} = -\frac{k_{c3}K_1(k_{c4}r_c)}{k_{c4}K_0(k_{c4}r_c)} \qquad (7)$$

Where
$$M_1 = -\frac{[\sigma_g k_{c1} I_0(k_{c1}r_a)/j\omega\varepsilon_0 - \varepsilon_1 I_1(k_{c1}r_a)]\dfrac{k_{c2}I_0(k_{c2}r_a)}{k_{c1}I_0(k_{c1}r_a)} + I_1(k_{c2}r_a)}{[\sigma_g k_{c1} I_0(k_{c1}r_a)/j\omega\varepsilon_0 - \varepsilon_1 I_1(k_{c1}r_a)]\dfrac{k_{c2}K_0(k_{c2}r_a)}{k_{c1}I_0(k_{c1}r_a)} - K_1(k_{c2}r_a)},$$

$$M_2 = \frac{k_{c3}}{k_{c2}}\frac{[I_1(k_{c2}r_b) - M_1 K_1(k_{c2}r_b)]}{I_0(k_{c2}r_b) + M_1 K_0(k_{c2}r_b)},\quad M_3 = \frac{I_1(k_{c3}r_b) - M_2 I_0(k_{c3}r_b)}{K_1(k_{c3}r_b) + M_2 K_0(k_{c3}r_b)}$$

III. Results of numerical calculation

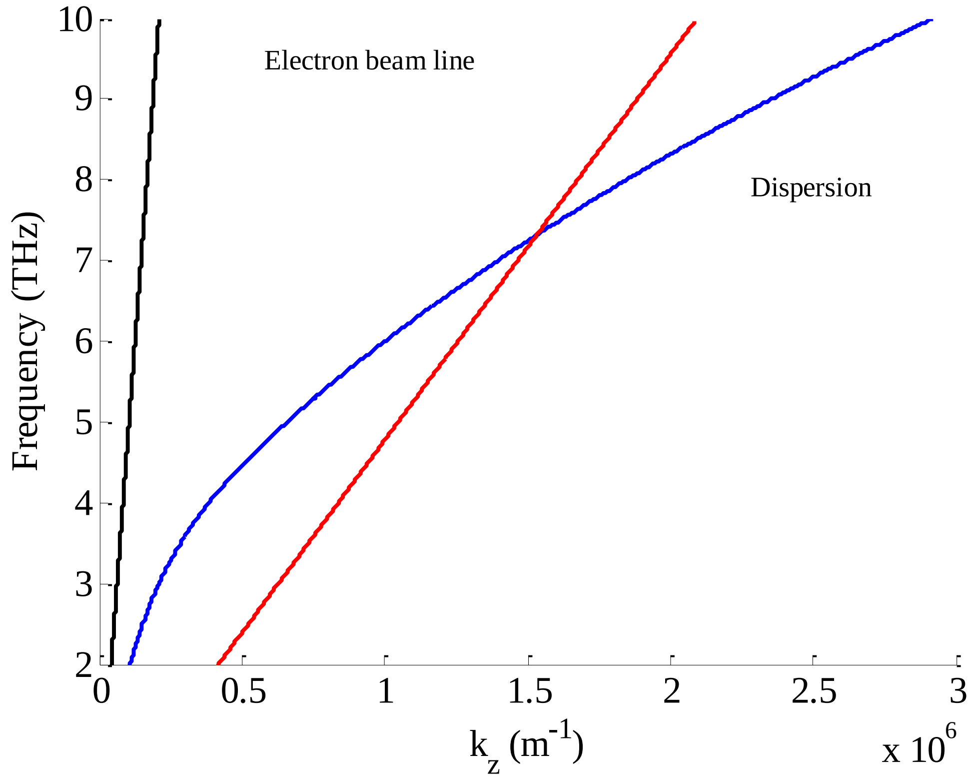

Fig. 2 Dispersion curve. The velocity of electron beam is 0.1c, c is the velocity of electromagnetic wave in the vacuum

The dispersion of graphene deposited on a dielectric rod is depicted in Figure 1. The structure parameters are: the radius of the dielectric rod $r_a = 3um$, $r_b = 3.2um$, and $r_c = 5.2um$. The permittivity of dielectric rod is $\varepsilon_s = 2$. Values of $T = 300K$, $\tau = 1.2ps$, and $\mu_c = 0.3eV$ are used. The electron beam line intersects with the dispersion line at frequency 7.2 *THz*. This intersection point is called working point. The physical mechanism of the energy transfer from electron beam to SPPs can be realized when the velocity of electron beam is moderately faster than the phase velocity of SPPs. The SPPs fields can modulate the velocity of the electron beam, while the decelerated electrons will loss part of energy to amplify the SPPs. With an experimental available current density 150 A/cm^2 of electron beam, the SPPs can be amplified. The gain can be calculated by

$$\begin{aligned} G &= 20\log\{\exp[\mathrm{Im}(k_z L)]\} \\ &= 8.686\,\mathrm{Im}(k_z L) \end{aligned} \quad (8)$$

L is the amplification length.

Therefore, by means of the electromagnetic theory which is quite similar to the space charge wave theory, when the velocity of electron beam is moderately faster than the phase velocity of SPPs, the electron beam transfer energy to SPPs and amplify the SPPs.

Figure 3 shows the comparison of field distributions of the structure with and without the electron beam. This clearly indicates that the loss of SPPs propagation is compensated.

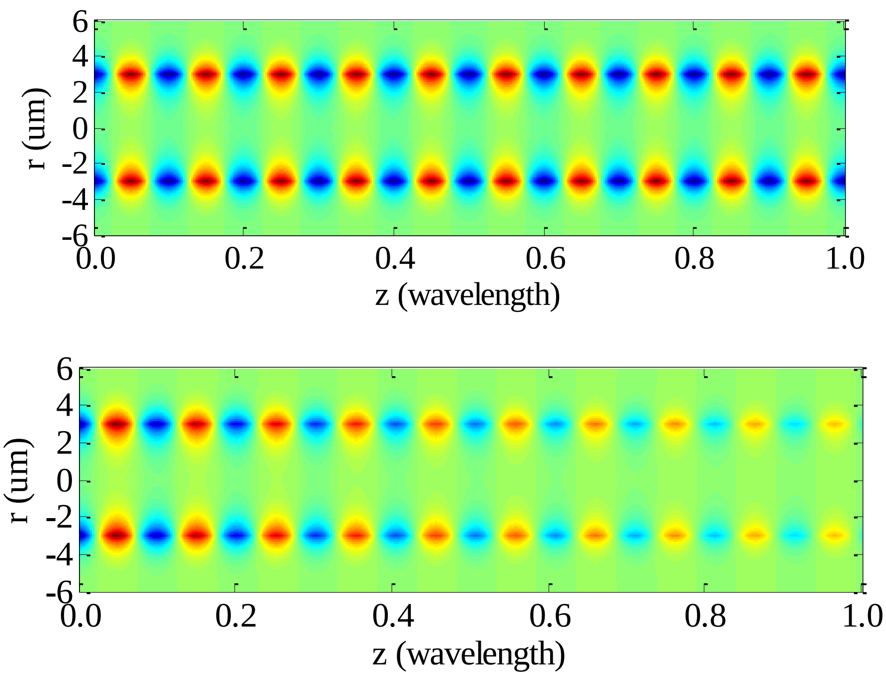

Fig. 3 SPPs field distributions of the structure with and without the electron beam

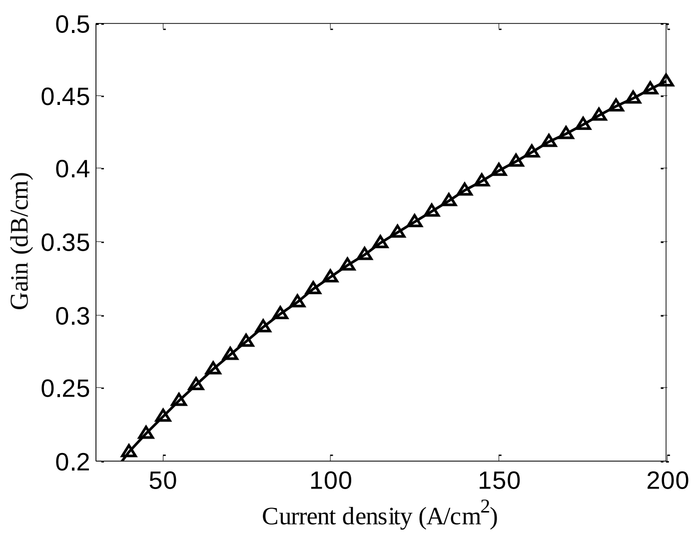

Fig. 4 Gain vs Current density

Figure 4 indicates the dependence of gain on the current density. The higher current density is, the larger gain.

IV. Conclusion

In this paper, a novel physical mechanism for building up amplifiers in the THz and higher frequency regime is presented. The electromagnetic theory of the SPPs excited by electron beam has been worked out, and it shows that when the velocity of the electron beam is moderately larger than the phase velocity of SPPs, the energy of electron beam transfers to SPPs leading to SPPs being amplified. This mechanism is just similar to the space charge wave theory, based on which the velocity of the fast space charge wave is moderately larger than the phase velocity of the wave, which will gain the energy from the beam and is amplified.

The results of the numerical simulation quite agree with that of the electromagnetic theory. Comparing to the THz and optical amplifiers appeared in references [13-17], the amplifier presented in this paper is much simpler and more practical.

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