# Nonlinear interaction of terahertz and optical waves in nitride films

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Abstract: It is considered the three-wave interaction between a space charge wave and two counterpropagating electromagnetic waves of terahertz or optical ranges in waveguides based on *n*-GaN or *n*-InN films. The waveguides are formed by the nitride films placed on a dielectric substrate. Space charge waves at the frequencies  $f \le 500 \text{ GHz}$  are amplified due to the negative differential conductivity, their finite widths lead to decrease of the amplification. It is possible to obtain amplification of the electromagnetic wave ~30 *dB* at the distances  $\le 100 \mu m$ . The input electromagnetic pulses of the durations  $\ge 100 \text{ ps}$  can be amplified by the three-wave interaction without distortions when the transverse width of the pulse is greater than 5  $\mu m$ .

**Keywords:** Negative differential conductivity, *n*-GaN, *n*-InN films, Three-wave interaction, Amplification of THz and optical pulses

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# **1. Introduction**

The electromagnetic radiation of terahertz range (THz) is widely used in spectroscopy, medicine, scanning, and environmental science [1]. Now the sources of the THz radiation are both electron tubes (gyrotrons, backward wave tubes), and lasers. These sources are not well compatible with the traditional millimeter wave devices. An alternative way can be realized with the amplification of electromagnetic waves (EMW) under three-wave resonant interactions in semiconductor films where the negative differential conductivity (NDC) occurs. This method is based on the transfer of amplification of a space charge wave (SCW) at the microwave range or the lower part of THz range to the electromagnetic wave at higher frequencies.

The amplification of SCW of the microwave range in *n*-GaAs films due to NDC was investigated for a long time [2]. It was also researched the transfer of amplification from SCW to EMW at higher frequencies under three-wave resonant interaction [3], so called the superheterodyne amplification [4].

Nonlinear three-wave resonant interactions in microwave, terahertz, and optical ranges were investigated in plasmas [5-8] and solids [9, 10]. In active plasma media, the explosive instability can be realized when all three interacting waves grow [5, 7, 8]. For the practical needs, in the active media, the transfer of the linear amplification of a low frequency wave to high frequency one is important [4, 7], i.e. the superheterodyne amplification. A preference of the

superheterodyne amplification is the using of relatively low levels of the electromagnetic pump wave at the intermediate frequency, which is utilized for the resonant wave coupling.

Nevertheless, the frequency range of amplified SCW in GaAs films is  $f \le 50$  GHz. The efficiency of amplification can be improved with using the new semiconductor materials like nitrides GaN and InN [11, 12]. These materials possess the following useful properties: the critical fields for observing NDC are high  $E_c \sim 100 \ kV/cm$ ; an extended frequency range for NDC  $f \le 500 \ GHz$ ; high temperature stability up till 500 K, NDC under high doping levels  $n_0 \le 10^{18} \ cm^{-3}$ .

In this article the amplification of SCW waves due to NDC is investigated in nitride (GaN, InN) films at the frequencies  $f \le 400 \text{ GHz}$ . The finite width of SCW leads to decrease of increments of amplification. For amplification of EM waves in optical and higher part of THz range, it is possible to use the mechanism of the superheterodyne amplification, which gives a possibility to increase the frequency of the amplified signal. The superheterodyne amplification of EM waves in waveguides based on nitride films is effective, namely, the values of 20 - 40 *dB* can be reached at the lengths of 10 - 100  $\mu m$ . An influence of the finite widths of EM waves has been taken into account.

# 2. Model and equations

It is considered *n*-GaN or *n*-InN film of submicron thickness placed on a dielectric substrate, as shown in Fig. 1. The nitride film is at  $0 \le x \le 2l$ , and a dielectric substrate is at  $x \le 0$ . Above at  $x \ge 2l$  there is either vacuum or a dielectric.



Fig. 1 Geometry of the problem. The region 0 < x < 2l is a nitride film, x < 0 is a dielectric substrate, and x > 2l is either vacuum or dielectric. The interacting waves are two EM waves  $(\omega_l, k_l)$ ,  $(\omega_2, k_2)$ , and a space charge wave  $(\omega_3, k_3)$ . The waves are nonuniform along *OY* axis. The bias constant electric field is aligned along *OZ* axis, and SCW propagates also along *OZ*.

The nitride film is the waveguide for EM waves when the condition is satisfied:  $\varepsilon_{2EM} > \varepsilon_{1EM}$ ,  $\varepsilon_{3EM}$ . Here  $\varepsilon_{1,2,3 EM}$  are corresponding permittivities in optical or THz ranges, which differ from their values in the microwave range due to the frequency dispersion [11-14]. In such a dielectric waveguide it is possible to realize the resonant three-wave interaction of the following waves localized along *OX* axis: forward EMW at the frequency  $\omega_l$  and the longitudinal wave number  $k_l$ , backward EMW  $\omega_2$ ,  $-|k_2|$ , and SCW  $\omega_3$ ,  $k_3$ . The resonant matching conditions are (shown in Fig. 1):

$$\omega_3 = \omega_1 - \omega_2, \quad k_3 = k_1 + |k_2| \tag{1}$$

The frequencies of EMW and SCW are of about:  $\omega_{l,2} \sim 10^{14} - 10^{15} \text{ s}^{-1}$ ,  $\omega_3 \approx 2 \omega_l \times (v_0 \varepsilon_{2EM})^{1/2} / c \sim 4 \times 10^{11} - 4 \times 10^{12} \text{ s}^{-1}$  ( $f_3 = \omega_3 / 2 \pi \sim 70 - 700 \text{ GHz}$ ) in the case of *n*-GaN film. Here  $v_0 \approx 2.4 \times 10^7 \text{ cm/s}$  is the velocity of SCW [11, 12].

The dynamics of SCW is described by the equations of motion of the electron fluid jointly with the Poisson equation for the electric field. At the frequency range  $f \le 400$  GHz, the simplest diffusion-drift equation can be applied:

$$\frac{\partial n}{\partial t} + div(\vec{v}(E)n - D\nabla n) = 0, \quad \vec{v} = \mu(|E|)\vec{E};$$

$$div(\varepsilon_0\varepsilon(x)\nabla\tilde{\varphi}) = -e(n - n_0), \quad \vec{E} = -\nabla\tilde{\varphi} + E_0$$
(2)

Here *n* is the electron concentration,  $\tilde{\varphi}$  is the potential of the variable electric field, *v* is the electron drift velocity,  $n_0$  is an equilibrium electron concentration, which is equal to the donor one; *D* is the diffusion coefficient,  $\mu(E)$  is the electron mobility, and  $E_0$  is a bias constant electric field. The data for nitrides GaN, InN are taken from [11, 12]. The coordinate frame is aligned along the crystalline axes. The lower indices 1, 2, 3 are related to the substrate, film, and the region over the film. The corresponding dielectric permittivities in the microwave range are  $\varepsilon_1$ ,  $\varepsilon_2$ ,  $\varepsilon_3$ .

## 3. Linear amplification of space charge waves in nitride films

Here the linear amplification of SCW due to NDC is considered. At the surfaces of the film the following boundary conditions are used for the density of the electric current j [2]:

$$j_x(x=0) = 0, \quad j_x(x=2l) = 0; \quad \vec{j} = e(\vec{v}(E)n - D\nabla n)$$
 (3)

Eqs. (1) and (2) have been linearized, and the solutions for the perturbation of the electron concentration  $\tilde{n}$  and for the variable part of the electric potential  $\tilde{\varphi}$  are searched as the travelling wave  $\sim exp(i(\omega t - k_z z - k_y y))$ . Now an attention is paid to the transversely nonuniform case  $k_y \neq 0$ . The dispersion equation for SCW  $k_z = k(\omega, k_y)$  has been obtained from the substitution of the solutions

within each partial region to the boundary conditions (3) and standard electric boundary conditions. In the case of spatial amplification of SCW, when a frequency  $\omega$  and a transverse wave number  $k_y$  are real ( $\omega > 0$ ), the longitudinal wave number is complex.  $k_z = k_z' + ik_z'' (k_z' > 0)$ ; within a certain frequency range there exists the imaginary part  $k_z'' > 0$ . The case of NDC is under consideration: dv/dE < 0.

The spatial increment  $k_z$ " depends essentially on the value of  $k_y$ . From the linearized equations it is possible to write down:

$$k_{z}^{\prime\prime}(\omega, k_{y}) \approx k^{\prime\prime}(\omega, 0) - gk_{y}^{2}; \quad g \approx 2l \frac{\omega_{M}}{\omega} \frac{\varepsilon_{2}}{\varepsilon_{1} + \varepsilon_{3}}$$
 (4)

Here  $v_0 = \mu(E_0)E_0$  is the constant part of the drift velocity, and  $\mu = \mu(E_0) = v_0/E_0$ ;  $\omega_M = en_0\mu/\varepsilon_0\varepsilon_2$  is the Maxwellian relaxation frequency. Therefore, a relatively small transverse nonuniformity of the beam of SCW  $\leq 5 \ \mu m$  results in the essential decrease of the amplification, because the value of the relaxation frequency  $\omega_M \sim 10^{13} \text{ s}^{-1}$  is high.

The results for *n*-GaN film with different thicknesses 2*l* and electron concentrations  $n_0$  for the bias electric field  $E_0 = 150 \ kV/cm$  are presented in Fig. 2 for the case of transversely uniform SCW  $k_y = 0$ . The curve 1 is for  $2l = 0.5 \ \mu m$ ,  $n_0 = 2 \times 10^{17} \ cm^{-3}$ ,  $\varepsilon_l = \varepsilon_3 = 3.9$  (SiO<sub>2</sub> above and below the film); the curve 2 is for  $2l = 0.5 \ \mu m$ ,  $n_0 = 2 \times 10^{17} \ cm^{-3}$ ,  $\varepsilon_l = \varepsilon_3 = 8.5$  (AlN); the curve 3 is for  $2l = 0.7 \ \mu m$ ,  $n_0 = 2 \times 10^{17} \ cm^{-3}$ ,  $\varepsilon_l = \varepsilon_3 = 8.5$  (AlN); the curve 3  $n_0 = 3 \times 10^{17} \ cm^{-3}$ ,  $\varepsilon_l = \varepsilon_3 = 8.5$  (AlN).



Fig.2. Dependence of spatial increment of amplification of SCW on frequency in different n-GaN films.

The amplification of transversely uniform SCW occurs in the frequency range  $f \le 400 \text{ GHz}$  and the maximum value of the spatial increment is  $k_z$  "  $\approx 2 \times 10^4 \text{ cm}^{-1}$  at the frequency  $f \approx 150 \text{ GHz}$ . When compared with *n*-GaAs films, in *n*-GaN films it is possible to obtain the amplification of SCW at essentially higher frequencies  $f \ge 200 \text{ GHz}$ . For *n*-InN films the results are similar to *n*- GaN ones, but the bias electric field can be lower,  $E_0 \sim 100 \ kV/cm$ , and the concentrations can be even higher than in GaN,  $n_0 \le 2 \times 10^{18} \ cm^{-3}$ .

# 4. The Equations for Three-Wave Interaction

The nonlinear interaction is due to the modulation of the permittivity in the optical range (at higher frequencies) by SCW and due to the ponderomotive action of EMW to SCW in the microwave range (at lower frequencies). In the case of moderate nonlinearity it is possible to describe this resonant interaction by means of slowly varying wave amplitudes [3, 4, 13, 14].

For EMW, the Maxwell equations are:

$$\nabla \times \vec{H} = \varepsilon_0 \varepsilon(x) \frac{\partial \vec{E}}{\partial t} + \vec{j}, \quad \vec{j} = e(n_0 + \tilde{n})\vec{v};$$

$$\nabla \times \vec{E} = -(1/\varepsilon_0 c^2) \frac{\partial \vec{H}}{\partial t}, \quad m^* \frac{\partial \vec{v}}{\partial t} \approx e\vec{E}$$
(5)

Here  $m^*$  is the effective electron mass,  $\tilde{n}$  is the variable electron concentration of SCW, and v is the high frequency electron velocity. It is investigated the interaction of transverse electric (TE) EM modes:  $E = E_y$ , the magnetic field is  $H = (H_x, 0, H_z)$ . The solutions of Eqs. (5) and Eqs. (1) for SCW and for EM waves are searched in the form:

$$E(x, y, z, t) = \frac{1}{2} (A_1(y, z, t) \exp(i(\omega_1 t - k_1 z)) + A_2(y, z, t) \exp(i(\omega_2 t - k_2 z)))F_1(x) + c.c.$$
(6)

$$\widetilde{n} = -\frac{i}{2}U(y,z,t)F_3(x)\exp(i(\omega_3 t - k_3 z)) + c.c.$$

Here  $A_{1,2}(y,z,t)$ , U(y,z,t) are slowly varying amplitudes for EMW and SCW,  $F_1(x)$ ,  $F_3(x)$  are linear transverse profiles of the waves. The pump wave is  $A_2$ ;  $A_1$  is the pulse under amplification,  $A_3$  is SCW, which is produced under the three-wave interaction. When using the orthogonality of waveguide modes [14], it is possible to get the coupled equations for slowly varying amplitudes:

$$\frac{\partial A_1}{\partial z} + \frac{1}{v_1} \frac{\partial A_1}{\partial t} + \frac{i}{2k_1} \frac{\partial^2 A_1}{\partial y^2} + \Gamma_1 A_1 = \alpha_1 A_2 U;$$

$$\frac{\partial A_2}{\partial z} - \frac{1}{v_2} \frac{\partial A_2}{\partial t} - \frac{i}{2|k_2|} \frac{\partial^2 A_2}{\partial y^2} - \Gamma_1 A_2 = \alpha_1 A_1 U^*;$$

$$\frac{\partial U}{\partial z} + \frac{1}{v_0} \frac{\partial U}{\partial t} - g \frac{\partial^2 U}{\partial y^2} - \Gamma_3 U = \alpha_3 A_1 A_2^*; \quad g \approx 2l \frac{\omega_M}{\omega} \frac{\varepsilon_2}{\varepsilon_1 + \varepsilon_3}$$
(7)

Here the dissipation coefficients for EM modes have been introduced  $\Gamma_1 = \Gamma_2$ ;  $\Gamma_3 > 0$  is the increment of spatial amplification of SCW that has been considered for case  $k_y = 0$ . One can see that the transverse nonuniformity of EMW leads to diffraction [9, 14], whereas the nonuniformity of SCW appears like the diffusion. SCW modulates the effective permittivity of EM waves, the influence of EM waves on SCW is ponderomotive. The coefficient  $\alpha_1$ , is real, whereas  $\alpha_3$  is imaginary;  $\alpha_{1,3} \sim S$ , where *S* is the overlap integral. The overlap integral is maximum for the symmetric dielectric waveguide (like SiO<sub>2</sub> – *n*-GaN (*n*-InN) –SiO<sub>2</sub> or AlN – *n*-GaN (or *n*-InN) – AlN. The expressions for the coefficients  $\alpha_{1,2,3}$  are:

$$\alpha_{1} = \frac{\omega_{p}^{2}}{4c^{2}k_{1}} \frac{S}{\int_{-\infty}^{+\infty} F_{1}^{2}(x)dx}; \quad \alpha_{3} = \frac{ie\mu_{d}k_{1}k_{3}}{m^{*}\omega_{1}^{2}} \frac{S}{\int_{0}^{2l} F_{3}^{2}(x)dx};$$

$$S = \int_{0}^{2l} F_{3}(x)F_{1}^{2}(x)dx \quad (8)$$

The values of the overlap integral *S* are presented in Fig.3.



Fig.3 The dependencies of overlap integrals on the frequency of EM wave  $\omega_l$ . Curve 1 is for GaN,  $2l = 0.5 \ \mu m$ ,  $n_0 = 2 \times 10^{17} \ cm^{-3}$ ,  $\varepsilon_l = \varepsilon_3 = 3.9$  (SiO<sub>2</sub> above and below the film); Curve 2 is for GaN,  $2l = 0.5 \ \mu m$ ,  $n_0 = 2 \times 10^{17} \ cm^{-3}$ ,  $\varepsilon_l = \varepsilon_3 = 8.5$  (AlN); Curve 3 is for  $2l = 0.7 \ \mu m$ ,  $n_0 = 2 \times 10^{17} \ cm^{-3}$ ,  $\varepsilon_l = \varepsilon_3 = 8.5$  (AlN); Curve 4 is for  $2l = 0.5 \ \mu m$ ,  $n_0 = 3 \times 10^{17} \ cm^{-3}$ ,  $\varepsilon_l = \varepsilon_3 = 8.5$  (AlN); Curve 5 is for InN,  $2l = 0.5 \ \mu m$ ,  $n_0 = 7 \times 10^{16} \ cm^{-3}$ ,  $\varepsilon_l = \varepsilon_3 = 8.5$  (AlN).

Note that the coefficient  $\alpha_3$  is complex. Our calculations have shown that the value of the overlap integral is maximum  $S \approx 1$  in the symmetrical dielectric waveguides for the fundamental EM modes. The transverse profiles are normalized as:

$$\int_{-\infty}^{+\infty} F_1^{2}(x) dx = \int_{0}^{2l} F_3^{2}(x) dx = 1 \ \mu m$$

It is possible to use the following structures of dielectric waveguides: the substrate  $SiO_2 - n$ -GaN (or *n*-InN) film–  $SiO_2$ ; AlN – *n*-GaN film – AlN. The symmetric dielectric waveguides are preferable, because they do not possess the frequency cut-off and provide the maximum values of overlap integral *S*.

# 5. Superheterodyne amplification of electromagnetic pulses

It is investigated the superheterodyne amplification of weak input pulses of EMW is at the carrier frequency  $\omega_l$ , i.e. the transfer of amplification of SCW due to NDC to EMW at higher frequency in the presence of the pump EMW at the frequency  $\omega_2$ . For this case the Eqs. (7) are added by the boundary conditions:

$$A_{1}(z = 0, y, t) = A_{10}\Phi(t) \cdot \Psi(y); \quad A_{2}(z = L_{z}, t) = A_{20};$$

$$U(z = 0, t) = 0 \quad (|A_{20}| \gg |A_{10}|); \quad , \qquad (9)$$

$$\Phi(t) = \exp(-(\frac{t - t_{1}}{t_{0}})^{2}); \quad \Psi(y) = \exp(-(\frac{y - L_{y}}{y_{0}})^{4})$$

where  $A_{10}$  is a maximum amplitude of the weak input pulse at the frequency  $\omega_l$ ,  $\Phi(t)$  is the temporal shape of the pulse,  $\Psi(y)$  is its transverse shape, and  $A_{20}$  is the constant amplitude of the EM pump wave. The temporal shape is chosen as Gaussian-like, while the transverse shape is bell-like with the half-width of about  $y_0$ .

The mechanism of the superheterodyne amplification is as follows [3, 4]. Because of the threewave interaction, the mixing of two EMW results in the generation of SCW at lower frequency. Then SCW is amplified in a medium with NDC. In that turn, in the output of the system the amplified EM wave appears at the frequency  $\omega_l$ .

The direct numerical simulations of Eqs. (7), (9) have shown the effective amplification of EM pulses of 50 *ps* - 20 *ns* durations. The simulations have demonstrated that it is possible to get the amplification of EMW 20 - 40 *dB* at the waveguide lengths  $\leq 100 \ \mu m$ ; the intensity of EM pump wave is  $\approx 10 - 100 \ kW/cm^2$ . Note that the intensity of the pump is moderate. The frequency of SCW is  $\omega_3 \approx 1.5 - 4 \times 10^{12} \text{ s}^{-1}$ , and the frequencies of EMW are  $\omega_{l,2} \approx 3 \times 10^{14} - 10^{15} \text{ s}^{-1}$ .

The results of simulations are presented in Figs. 4, 5, 6. The symmetric dielectric waveguide  $SiO_2 - n$ -GaN - SiO<sub>2</sub> is considered where the maximum values of the overlap integral are realized, shown in Fig. 3, curve 1. The electron concentration is  $n_0 = 2 \times 10^{17} \text{ cm}^{-3}$ , the thickness of the film is  $2l = 0.5 \ \mu m$ , and the permittivities in the optical range are  $\varepsilon_{I EM} = \varepsilon_{3 EM} = 2$  (SiO<sub>2</sub>);  $\varepsilon_{2} EM = 5.8$ . The frequencies of the EMW are  $\omega_l \approx 6.5 \times 10^{14} \text{ s}^{-1}$ ; the corresponding resonant frequency of SCW is  $\omega_3 \approx 2.2 \times 10^{12} \text{ s}^{-1}$  ( $f_3 \approx 350 \text{ GHz}$ ); the permittivities in the millimeter wave

range of the media are  $\varepsilon_1 = \varepsilon_3 = 3.9$ ,  $\varepsilon_2 = 9.7$ . At the plots for  $A_1$  the unity corresponds to the EM wave intensity 10 *MW/cm*<sup>2</sup>. The pump intensity at the frequency  $\omega_2$  is 16 *kW/cm*<sup>2</sup>. The length of the waveguide is  $L_z = 80 \ \mu m$ .

In Fig. 4 the amplification of a longer input EM pulse of 100 ps duration is investigated. The transverse nonuniformity of the input pulse is not important when its transverse width is greater than 10  $\mu$ m. At smaller widths the amplification decreases. Nevertheless, the narrower pulses of the widths 5 – 10  $\mu$ m also can be amplified while preserving their shapes.



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Fig.4 Results of simulations of superheterodyne amplification for a longer pulse  $A_1$ . Part a) is the input pulse  $A_1$ , b) is the output pulse  $A_1$  (amplified), transversely uniform pulses, c) is the amplification coefficient for EM wave  $A_1$ ; Parts d), e) and f), g) are the output pulse  $A_1$  (amplified) and spatial distribution at the maximum output value, transversely nonluniform pulses. For the parts d), e) the transverse width is  $y_0 = 10 \ \mu m$ , for the parts f), g)  $y_0 = 5 \ \mu m$ .

Results of simulations of superheterodyne amplification of a shorter input pulse  $A_1$  of 50 ps duration are presented in Fig. 5. One can see that the broadening of the amplified pulses occurs there. Therefore, the lower limit of the input EM pulse duration is of about 100 ps.





Fig. 5 The case of a shorter input pulse. The parameters of films are the same as for the longer case.

The dynamic range for the input amplitudes of EM pulses is quite high and can reach the values  $10^4 - 10^5$ . In the undimensional units, the input values of  $A_1$  can be  $10^{-14} - 10^{-9}$  for the used parameters. When the input pulses possess higher amplitudes, the nonlinear regime of amplification takes place. The exhaustion of the pump wave  $A_2$  occurs. This results in the decrease of the amplification of the pulse  $A_1$  and its distortion. These data are shown in Fig.6.





Fig. 6 Amplification of EM pulse of a higher amplitude, transversely uniform case. Part a) is the input pulse  $A_1$ , b) is the output pulse  $A_1$  (amplified), and c) is the amplification coefficient for different moments of time.

Thus, for amplification of EM waves in optical and higher part of THz range it is possible to use the mechanism named superheterodyne one, which leads to transfer the amplification from lower frequencies, millimeter wave range, to the signals at higher frequencies. This superheterodyne amplification of EM waves in dielectric waveguides based on nitride films is effective, and the values of the amplification coefficients are 20 - 40 *dB* and can reach at the lengths of 20 - 100  $\mu m$ .

#### 6. Conclusions

Amplification of space charge waves at the frequencies  $f \le 400 \text{ GHz}$  in the waveguides based on *n*-GaN or *n*-InN films can reach the values of 20 - 40 *dB* at the distances of 20  $\mu m$ . The transverse nonuniformity of SCW beams leads to decrease of increments of amplification and seems like wave diffusion. Therefore the transverse widths of the films should be greater than 5  $\mu m$ .

The transfer of amplification under three-wave resonant interaction from the space charge wave to the electromagnetic wave at higher frequencies  $\omega_l \sim 10^{14} - 10^{15} \text{ s}^{-1}$  can be realized in dielectric waveguides based on nitride films of ~0.5  $\mu m$  thicknesses with electron concentrations ~2×10<sup>17</sup> cm<sup>-3</sup>. This superheterodyne amplification can reach 20 - 40 dB at the lengths 20 - 100  $\mu m$ . The input EM pulses can be of subnanosecond durations. The superheterodyne amplification can be realized under moderate values of the EM pump 10 – 100 kW/cm<sup>2</sup> at the intermediate frequency.

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