The Theoretical Study on THz Frequency Comb Generation via Optical Rectification

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Abstract: In this paper, we proposed a method for THz comb generation via surface-emitted optical rectification (OR) of ultra-short laser pulse in periodically poled lithium niobate (PPLN). The mechanism of this phenomenon is spectral interference between early and late THz pulses emitted by one fs-pulse. The generation of THz comb was analyzed both in frequency- and time-domain based on radiating antenna model. The figures of spectrum and waveform of the calculated electric field were presented. Our calculation indicated that THz combs generated by this method cover a large bandwidth and have a wide free spectral range.

Keywords: Optical Rectification, Frequency Comb, Terahertz

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1. Introduction

The frequency comb technique has attracted considerable interest over recent years, because of its application in frequency metrology and precision spectroscopy. A “frequency comb” is a kind of electromagnetic wave, which has a spectrum with a discrete, regularly spaced series of sharp lines. It can be used as a ruler to measure the frequency of an unknown electromagnetic wave with extraordinary high precision.

The recent studies on the frequency comb technique are mainly in optical region. An fs-pulse train generated by a mode-locked laser has a comb-like spectrum [1, 2]. Another method for optical frequency comb generation is superimposing a microwave frequency onto an optical carrier frequency by an electro-optic phase modulator [3~5]. Besides, optical frequency comb can also be generated through cascaded optical parametric oscillation and four-wave mixing [6, 7]. The combs mentioned above mainly cover from the visible to near-infrared region. Because of the outstanding spectral properties, it has significant advances in the measurement of atomic spectrum, the observation of energy level hyperfine structure, the determination of fundamental constants and also the generation of all-optical atomic clock. Thus it can be widely used in optical frequency metrology, precision spectroscopy, all-optical atomic clock and frequency synthesis.

In THz region, frequency comb technique is still immature. Recently, a THz comb generated from a photoconductive emitter excited by an fs-pulse train (fs optical comb) has been reported [8, 9]. When an fs optical comb excites a photoconductive material, it is down converted to the terahertz region without any change to the frequency spacing. So the generated THz wave by this method also has a comb-like spectrum. This technique extended the concept of frequency comb to the THz region, so the identification with high-precision of molecules of interest with fingerprints in THz region, such as pollution, explosives and biological molecules, can be achieved. And terahertz precision spectroscopy can get long-term development with this technique. But in this method the THz comb is generated by an optical frequency comb and merely covers a range about 1*THz*.

In this paper, we proposed a method for THz comb generation via surface-emitted optical rectification of one fs-pulse in a PPLN. The pump source in our method is a single laser pulse rather than an optical frequency comb. The principle of THz comb generation via OR was theoretically analyzed. The mechanism of this phenomenon can be explained by spectral interference between early and late THz pulses emitted by one fs-pulse. The expressions of electric field of the generated THz comb both in frequency- and time-domain were obtained. The figures of spectrum and waveform of calculated THz comb were presented. Our calculation indicated that THz combs generated by this method cover a large bandwidth and have a wide free spectral range.

2. Theory

Theory of generation of narrow-band THz wave by surface-emitted optical rectification of fs-pulses in PPLN has been discussed in detail by Avetisyan et al. [10, 11]. The principle of THz comb generation in this method is based on radiating antenna model. When an ultra-short laser pulse is incident on a nonlinear crystal, a second-order nonlinear polarization $P_{NL}^{(2)}$ is created . The rectified polarization can be written as Eq.(1) [10]:

$$P_{NL}^{(2)}(t)=\varepsilon_0 d(x)E_0^2\exp(-\frac{2y^2+2z^2}{r_0^2})\exp[-\frac{(t-x/u)^2}{\tau_L^2}] \tag{1}$$

With the radiating antenna model:

$$\vec{J}(\omega)=i\omega\overrightarrow{P_{NL}^{(2)}}(\omega) \tag{2}$$

$$\vec{A}(\omega)=\frac{\mu_0}{4\pi}\int\frac{\vec{J}(\omega)\exp(-i\vec{k}\cdot\vec{r})}{r}dV \tag{3}$$

$$\vec{E}(\omega)=-i\omega\vec{A}(\omega) \tag{4}$$

The expression of the rectified electric field in frequency domain was derived as:

$$E(\omega)\propto\omega^2\exp(-\frac{\tau_L^2+\tau_a^2}{4}\omega^2)\sum_{i=-N}^{N}\int_{\frac{2i-1}{2}l_c}^{\frac{2i+1}{2}l_c}(-1)^i d_{eff}\cos(\frac{\omega\Delta n}{c}x)dx \tag{5}$$

There is a key factor in the expression, which is a series of integrals over individual domains (with coherent length l_c), because the nonlinear coefficient of PPLN: d_{eff} changes its sign in neighboring domains. We use trigonometric transformation to simplify the series into one single term. And we can obtain a simplified expression:

$$|E(\omega)| \propto d_{eff}\frac{c}{\Delta n}\omega\exp(-\frac{\tau_L^2+\tau_a^2}{4}\omega^2)\left|\frac{2\sin(\frac{l_c\Delta n}{2c}\omega)\cos(65\frac{l_c\Delta n}{2c}\omega)}{\cos(\frac{l_c\Delta n}{2c}\omega)}\right| \tag{6}$$

After inverse Fourier transformation of the electric field in frequency domain, the expression of the rectified electric field in time domain is obtained as:

$$E(t) \propto \sum_{k=-N}^{N}(-1)^k[f(t-\frac{\Delta n}{c}\frac{2k-1}{2}l_c)-f(t-\frac{\Delta n}{c}\frac{2k+1}{2}l_c)] \tag{7}$$

where

$$f(t)=\frac{t}{\tau_{eff}^3}\exp(-\frac{t^2}{\tau_{eff}^2}) \tag{8}$$

With these expressions, we can get different patterns of spectrum by choosing different parameters: the poling period of the crystal $\Lambda = 2l_c$ and the effective pulse duration τ_{eff}, which relates to the time duration of the in-put fs-pulse τ_L and the beam waist r_0. In previous works [10, 11], the output THz wave is narrowband with one center frequency. While $\Lambda = 780\mu m$ and $\tau_{eff} = 200fs$, the radiated field has a spectrum that consists of a discrete, regularly spaced series of sharp lines (Fig.1). The lines are in THz region, so this spectrum is a THz frequency comb.

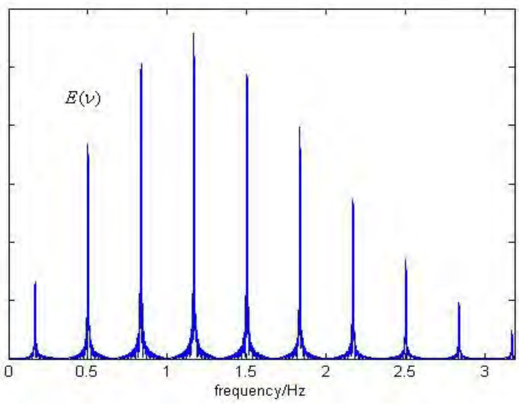

Fig.1 The spectrum of a THz comb ($\Lambda = 780\mu m$, $\tau_{eff} = 200fs$).

As seen from Fig.1, this THz comb covers up to $3THz$. The spectral resolution (bandwidth of each component) in the comb-like spectrum can be calculated based on Eq.(6). For the case N = 32 and $l_c = 390\mu m$, the bandwidth of one frequency component $\Delta\omega$ is $18.5GHz$. The free spectral range (interval between two adjacent components) is $0.33THz$, which is much wider than $\Delta\omega$. This THz comb will be an ideal source for wavelength division multiplexing in THz wireless communication system, for it can be separated into its individual components easily, for demultiplexing and modulation of each channel.

We can also get the corresponding temporal form (Fig.2). To see it clearly, we focus on part of the wave (Fig.3). As seen from the figures, the whole wave consists of a train of single cycle pulses, and the polarities of every two adjacent pulses are opposite to each other. The number of pulses equals to two times the number of poled periods of the crystal. The temporal width of the THz wave is $T = n_g L/c$, which is the traveling time of the pump pulse passing through the crystal. And the time delay between the two pulses $t_d = l_c n_g/c$, which is the traveling time of the pump pulse passing through one domain.

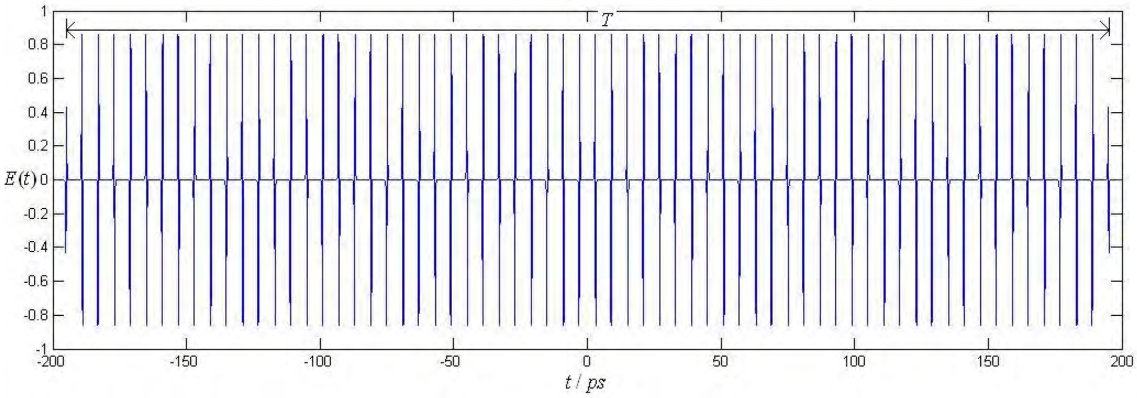

Fig.2 Temporal form of THz comb.

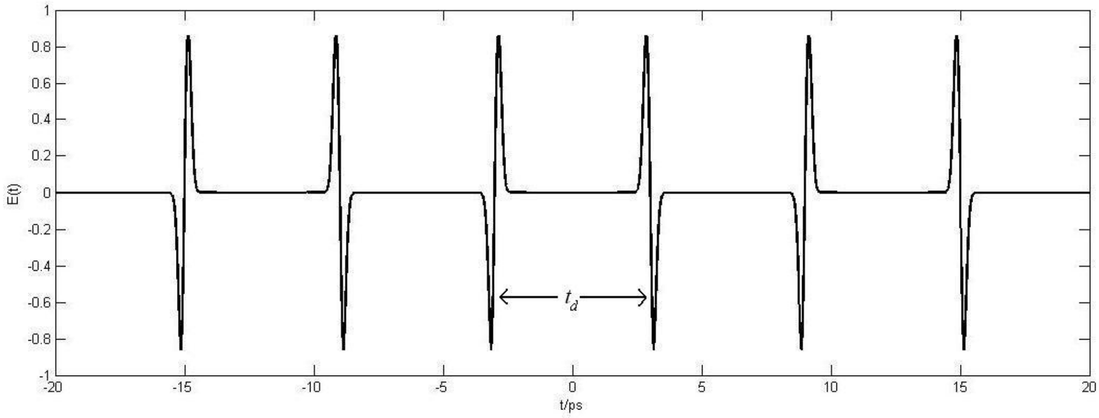

Fig.3 microscope of Fig.2.

As is analyzed in previous articles [11], a THz radiation with multi-cycles and narrow-band can be generated from a PPLN. One cycle of the radiation corresponds to one poled period of the crystal. Every pulse generated from the corresponding domain (coherent length l_c) has a wideband spectrum in frequency domain. The characteristic of the spectrum is determined by the envelop factor $G(\omega)$, which is affected by the parameter τ_{eff}. THz pulses radiated from different domains interfere with each other. This process is spectral interference between early and late pulses. The frequencies can output if they satisfy the constructive interference condition:

$$\omega = (2k+1)\frac{\pi c}{l_c \Delta n}, k \in N \tag{9}$$

which is determined by factor F. If the parameters (period of PPLN crystal and effective pulse width) are selected properly, high-order harmonic ($3\omega_0$, $5\omega_0$, …) can gain and output. Then there will be several spikes in the spectrum, and the frequency comb is obtained.

3. Summary

In this paper, we proposed a new method for THz frequency comb generation via surface-emitted OR of fs-pulse in PPLN. We theoretically discussed the principle and process of the generation in detail. The mechanism of this phenomenon was explained by spectral interference between early and late THz pulses emitted by one fs-pulse. Based on the radiating antenna model, analytical equations for electric field of THz radiation were derived both in frequency- and time-domain.

In contrast to previous theoretical analysis [10], we use trigonometric transformation to simplify the series into one single term. Finally, a brief expression: $|E(\omega)| \propto G(\omega)F(\omega)$ was obtained. We can obtain different patterns of spectrum by choosing different parameters: the poling period of the crystal and the effective pulse duration. As the factor $F(\omega)$ is a periodical function with several spikes, if the parameters are designed properly, a comb-like spectrum that consists of a discrete, regularly spaced series of sharp lines in THz region can be observed. The temporal form of electric field was also presented. The waveform consists of a train of single cycle pulses, and the polarities of every two adjacent pulses are opposite to each other.

The THz comb via this method covers a range up to 3 *THz*. And it is with modes spaced by several hundreds of gigahertz, so this comb may be easily divided into its constituents for access and control of individual modes. Thus it is especially useful in wavelength division multiplexing (WDM) optical communication system.

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