Single-Cavity Gyromultipliers

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Abstract: Several new configurations of self-exciting gyro-multipliers are proposed. These schemes allow concurrent excitation of radiation at fundamental and harmonic frequencies in a single resonator. It is shown that such an approach simplifies the experimental setup and promises high device efficiency. Experimental proposals based on some of the schemes are presented.

Keywords: Cyclotron resonance masers, Gyrotrons, Frequency multiplication

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1. Introduction

Cyclotron resonance masers (CRMs, see e.g. [1-5]) have proved to be efficient and powerful sources of coherent electromagnetic radiation in the centimeter, millimeter, and sub-millimeter wavelength ranges. The most widespread and perfect types of CRMs are gyrotrons and gyroklystrons, operating at low (fundamental and second) cyclotron harmonics. For instance, fundamental-harmonic millimeter-wavelength gyrotrons with weakly relativistic electron beams produce huge power of up to 1 MW in the continuous wave (CW) regime with high efficiency (see e.g. [5]). Powerful gyrotrons have been successfully realized also in a number of sub-millimeter experiments [6-10] including the pulsed oscillator which recently achieved a wavelength of 0.3 mm [10]. However, the necessity to use very strong magnetic fields in this very promising part of electromagnetic spectrum (over 35 T at the wavelength of 0.3 mm, when operating at the fundamental harmonic) essentially makes such sources impractical for many potential users. To reduce the required magnetic field an alternative approach uses a Doppler upshift to reduce the required magnetic field, as in the cyclotron autoresonance maser (CARM). This approach has been successful [11,12] but has the disadvantage of requiring less convenient, significantly higher energy, high quality electron beams.

The magnetic field in a gyrotron can certainly be significantly diminished if operation at a high (n>2) cyclotron harmonic is used, but it is very difficult to achieve this in traditional gyrotrons with tubular weakly/moderately relativistic electron beams because of the dramatic weakening of the electron-wave coupling [1,2] and parasitic mode excitation at low harmonics. A significantly more efficient and selective interaction at high harmonics can be provided in the so-called Large Orbit Gyrotrons (LOGs, [13-17]) and in gyromultipliers [18-25].

In LOGs, selective operation at the 3^{rd}-5^{th} harmonics has already been demonstrated at short wavelengths and a minimum wavelength of 0.7 mm has been achieved due to using axis-encircling electron beams [13-17]. In gyromultipliers, a relatively low-frequency (LF) signal provides electron modulation and bunching at the frequency of the operating wave and also simultaneously at its harmonics, which occurs due to the nonlinear properties of the

electron beam. Because of this preferential frequency imposed on the electron beam, high-frequency (HF) radiation at a high cyclotron harmonic can be selectively excited [18-25]. The most selective and efficient frequency multiplication can be obtained if, like in LOGs, one uses axis-encircling electron beams. It is important that for self-exciting multipliers only the starting current for the LF harmonic should be exceeded and the operating current can be significantly smaller than for high-harmonic gyrotrons, for which the problem of the starting current is rather severe. Certainly, at smaller currents the HF power from gyromultipliers will be lower than from high-harmonic gyrotrons, but it is nevertheless quite sufficient for most applications, and the advantages of the formation of low-current beams may often overbalance this lack of power.

The LF wave in a gyromultipler may either be induced internally or injected externally. A self-exciting gyro-multiplier, which needs no external RF source, can be often more attractive at short wavelengths. In such a device, the electron beam excites both (LF and HF) waves. In Sect. II of this paper, two possibilities for the realization of self-exciting gyro-multipliers are compared: a two-cavity scheme with self-exciting LF cavity and a scheme of combining both the LF and HF oscillators in a single cavity. In Sections III and IV, variants of single-cavity two-wave gyro-oscillators with uniform and non-uniform magnetic fields are studied.

2. Two-cavity scheme of gyro-multiplier

The most evident variant of the gyro-multiplier is a two-cavity klystron-like scheme with an external LF source (Fig. 1) [18-25]. In the first section, the electron beam is in resonance with the low-frequency wave at the fundamental cyclotron harmonic:

$$\omega_{LF} \approx \Omega + h_{LF} v_{\|} \tag{1}$$

The second resonator is adjusted to the resonance with the HF wave at the *n*-th cyclotron harmonic:

$$\omega_{HF} \approx n\Omega + h_{HF} v_{\|} \tag{2}$$

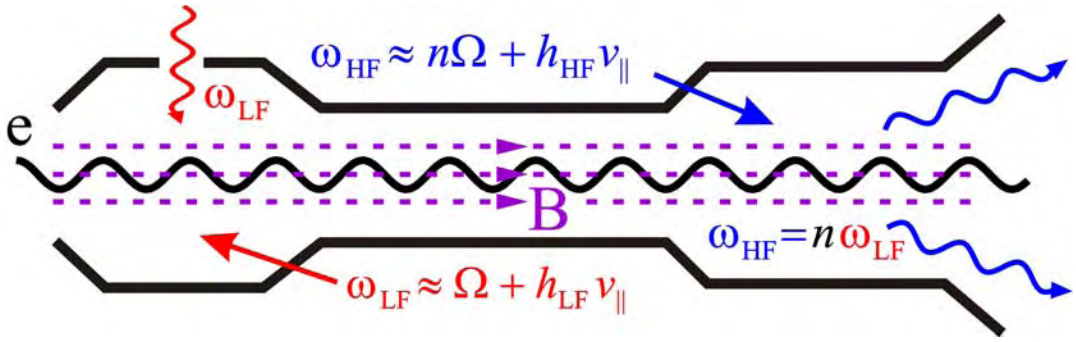

Fig. 1 Schematic of a two-cavity gyromultiplier with an external low-frequency RF signal.

Here ω_{LF} and ω_{HF} are the frequencies of the LF and HF waves, h_{LF} and h_{HF} are their longitudinal wavenumbers, $v_{\|}$ is the axial electron velocity, and $\Omega = eB/mc\gamma$ is the electron gyro-frequency with the relativistic Lorentz factor $\gamma = 1/\sqrt{1-(v/c)^2}$. In the first cavity the electron beam is modulated by the LF wave at its frequency, ω_{LF}. In the drift region between

the cavities the modulation leads to creation of electron-current components at all harmonics of this frequency. In the second cavity the bunched beam can excite the HF wave at the *n*-th cyclotron harmonic, if the cyclotron harmonic number coincides with the number of the electron-current harmonic:

$$\omega_{HF} = n\omega_{LF} \tag{3}$$

In the case of a uniform magnetic field, conditions (1-3) automatically make the divisibility of the longitudinal wavenumbers necessary,

$$h_{HF} \approx nh_{LF} \tag{4}$$

For example, if the gyrotron-type ($h_{LF} << \omega_{LF} / c$) resonance is provided in the first cavity, the same type of resonance should be provided in the second cavity.

Certain selectivity rules for the transverse spatial structure of the HF wave exist as well, but they depend on specific geometries of both the cavity and the electron beam. In particular, in the case of the most convenient cavities with circular transverse cross-sections, the ratio between the azimuthal indices of the HF and LF modes should be equal *to the frequency multiplication number,* $m_{HF} = nm_{LF}$. Especially strong mode selectivity takes place for an axis-encircling electron beam used in LOGs, when the transverse motion of all the electrons corresponds to gyrating around the axis of the circular cavity. In this case, both (HF and LF) modes can be excited only at cyclotron harmonics coinciding with azimuthal indices of the modes [2,13-17].

A simple analysis of the described scheme (Fig.1) can be carried out by the means of a general theory of RF devices based on inertial electron bunching. For this situation, electron motion in the field of a wave [26] is described by the well-known asymptotic equations:

$$\frac{dw}{dz} = \kappa \,\mathrm{Im}(ae^{i\theta}), \frac{d\theta}{dz} = \nu w - \Delta, w(0) = 0, \theta(0) = \theta_0 \tag{5}$$

which are applicable if the relative change in electron energy, *w*, under the influence of the RF field is sufficiently small. Here θ is the relative phase between the electron and the wave which can be an arbitrary value, Δ is the resonance mismatch, κ and ν are the factors of electron-wave coupling and inertial bunching, respectively, *a* is the normalized complex amplitude of the RF field, and *z* is the normalized longitudinal coordinate. For an unbunched electron beam entering the first resonator, the initial phases of particles, θ_0, are distributed uniformly over the interval $[0,2\pi)$.

The degree of bunching at the *n*-th harmonic of the LF wave frequency is described by the normalized density of the RF electron current,

$$\rho_n = \left\langle e^{-in\theta} \right\rangle \tag{6}$$

where <...> denotes averaging over the whole ensemble of particles. Using the approximation of a short input cavity, $a(z) = a_0 \delta(z) L_0$ (here L_0 is the length of the first cavity and $\delta(z)$ is the delta-function), one obtains the following solution for the electron energy and the phase in the drift region between the cavities:

$$w(z>0)=\kappa a_0 L_0 \sin\theta_0$$
$$\theta(z>0)=\theta_0-\triangle z+zv\kappa a_0 L_0 \sin\theta_0 \quad (7)$$

yields the following expression for ρ_n:

$$\rho_n = J_n(\chi nz), \chi = v\kappa a_0 L_0 \quad (8)$$

where J_n is the Bessel function of the n-th order and χ is the klystron bunching parameter. The dependence of current harmonics on the coordinate is shown in Fig. 2. It is important that higher harmonics reach saturation earlier than the lower harmonics.

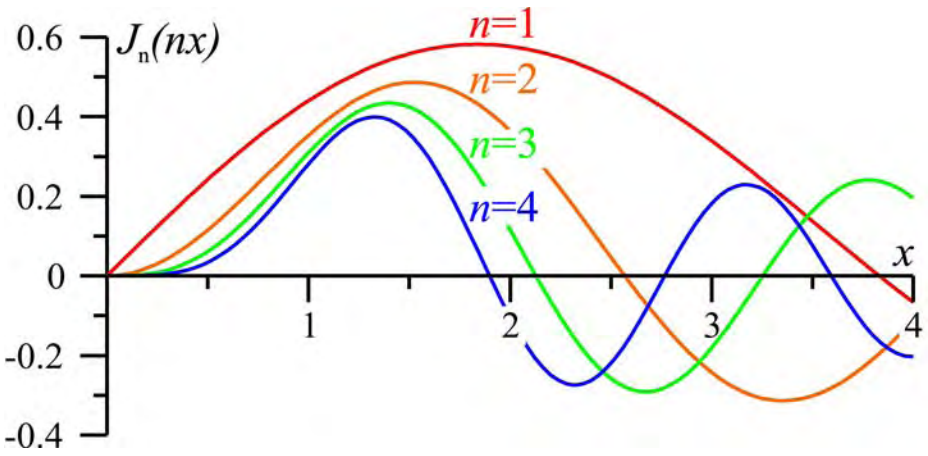

Fig. 2 Axial distribution of harmonics of the electron current in a klystron.

In the second cavity, the existence of the current harmonic ρ_n results in radiation at the n-th harmonic of the modulating frequency. In principle, if the amplitude of the operating wave inside the cavity is close to its optimal value, then the efficiency of radiation at any harmonic can be rather high. The maximum of the orbital efficiency, $\eta_\perp$, which is the averaged part of the transverse electron energy spent in radiation, varies from 60% for n =2 to 20% for n =5 [19]. However, in the case of a high number of the operating cyclotron harmonic, it is quite problematic to provide the optimal RF-wave amplitude. Actually, the electron-wave coupling factor, κ, decreases fast with the increase of the cyclotron harmonic number. Correspondingly, the optimal value of the RF amplitude increases, $a \propto 1/\kappa$. Thus, in order to provide the optimal conditions of the electron-wave interaction, one should provide a high enough value of either the electron current, or the Q-factor of the operating cavity. However, the latter is strictly limited by the Ohmic Q-factor of the cavity. Thus, at moderate values of the electron current, the RF amplitude proves to be significantly smaller than the optimal value, and this fact leads to relatively low electron efficiencies.

Let us illustrate this fact by the following estimates. In the case of the axis-encircling electron beam, the interaction efficiency is determined by the normalized current parameter [19]

$$\hat{I}=8\frac{eI}{mc^3}\frac{\lambda_{HF}Q_2}{L_2N_2}\left(\frac{n^n}{2^n n!}\right)^2\frac{\beta_\perp^{2(n-2)}}{\beta_\parallel} \quad (9)$$

where I is the electron current, L_2 and Q_2 are the length and the Q-factor of the second cavity, $N_2 = (\nu_{n,p}^2 - n^2) J_n^2(\nu_{n,p})/2$ is the norm of the HF mode (here ν is the p–th zero of the derivative of the Bessel function, $J_n'(\nu_{n,p}) = 0$), λ_{HF} is the HF wavelength, $\beta_{\perp,\|} = v_{\perp,\|}/c$ are the transverse and longitudinal electron velocities normalized to the speed of light. For achieving efficient radiation, one should provide quite a large value of the normalized current parameter, $\hat{I} \approx 0.1$ (actually, it corresponds to the electron current close to the starting current for the second cavity).

Let us assume that the quality of the operating mode is equal to the minimum diffraction Q-factor, $Q = 8\pi \left(\frac{L_2}{\lambda_{HF}} \right)^2$ and find the optimal electron current for interaction at the fifth cyclotron harmonic in the case of a typical gyrotron beam of voltage 60 kV and pitch-angle $\beta_\perp / \beta_\| = 1$:

$$I \approx \frac{N_2}{4} \frac{mc^3}{e} \frac{\lambda_{HF}}{L_2} \tag{10}$$

Thus, if the length of the cavity amounts to several wavelengths, then too high electron currents of the kA level are required to provide the optimal electron-wave interaction. If the electron current is significantly lower (for example, a few amperes), then in the low-current approximation the orbital electronic efficiency is described by the formula

$$\eta_\perp = 16 \frac{\hat{I}}{\beta_\|} \left(\frac{L_2}{\lambda_{HF}} \beta_\perp \rho_n \right)^2 \tag{11}$$

It means that with the same assumptions about the beam parameters the orbital efficiency is determined by the expression

$$\eta_\perp = \frac{2}{N_2} \frac{I}{mc^3/e} \left(\frac{L_2}{\lambda_{HF}} \right)^3 \rho_5^2 \tag{12}$$

In the case of an electron current of the order of 1A this value is as small as 0.3%. Gyromultiplier efficiency can also be noticeably reduced due to Ohmic losses, inhomogeneity of the HF current ρ_n inside the cavity, the use of a high transverse HF mode with a large norm, and so on. Thus, the expected efficiency of any gyromultiplier should be compared with the rather low estimate given by (11).

In principle, the klystron-like scheme can be used as a self-exciting gyro-multiplier, which requires no external source of the LF signal; in this case, the first cavity represents a self-exciting LF auto-oscillator of a certain type, e.g. gyrotron, gyro-BWO or resonant gyro-TWT. Let us consider a gyro-BWO as the simplest example of the first LF oscillator (Fig. 3) which is characterized by a minimal number of parameters and in addition allows a frequency tuning. In this case, the first LF oscillator is described by the system of equations (5), supplemented with the equation for the complex amplitude of the opposite (backward) wave (these equations coincide with the ones for a Cherenkov BWO [27]):

$$\frac{da}{dz} = --iG\rho_1, a(L_0) = 0 \tag{13}$$

where G is the factor of wave excitation proportional to the beam current. It is assumed that the electron beam enters the cavity at $z = 0$. Having introduced the normalization to the Pierce parameter $C = \sqrt[3]{v\kappa G}$

$$\hat{z} = Cz, \hat{w} = wv/C,\ \hat{a} = aC/G, \hat{\Delta} = \Delta/C \tag{14}$$

one transforms Eqs. (5) and (13) to the following form:

$$\frac{d\hat{w}}{d\hat{z}} = \mathrm{Im}(\hat{a}e^{i\theta}), \frac{d\theta}{d\hat{z}} = \hat{w} - \hat{\Delta}, \frac{d\hat{a}}{d\hat{z}} = -i\rho_1 \tag{15}$$

with boundary conditions

$$\hat{w}(\hat{z} = 0) = 0,\quad \theta(\hat{z} = 0) = \theta_0,\quad \hat{a}(\hat{z} = \hat{L}) \tag{16}$$

Thus, the regime of BWO operation is determined by only one parameter, the normalized length of the generator, $\hat{L} \sim L_0\sqrt[3]{I}$. The oscillator is excited if the length exceeds the starting limit, $\hat{L}_{st} = 1.98$. Simulations show, that if the normalized length, $\hat{L}$, is not too close to the starting value, $\hat{L}_{st}$, then the first harmonic of the electron current reaches a maximum inside the LF oscillator, $\hat{z} < \hat{L}$ (Fig.3). As for higher harmonics, their behavior is very similar to the case of the klystron model, i.e. they become saturated earlier than the first harmonic.

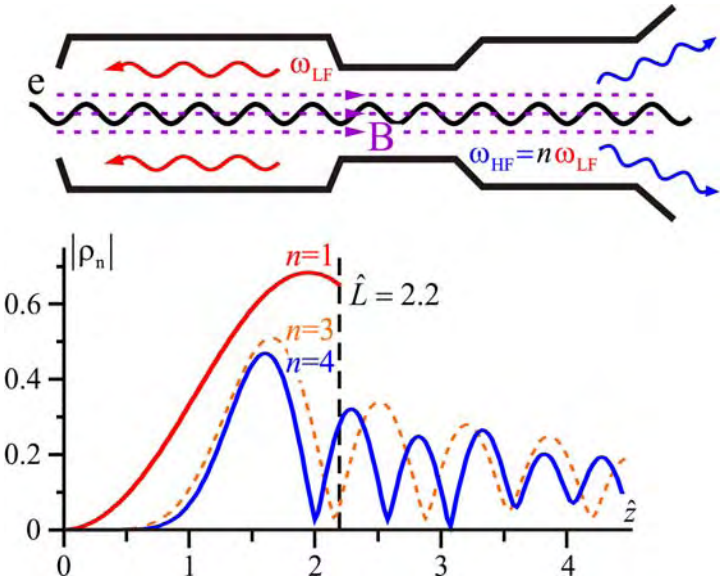

Fig. 3 Schematic of a two-cavity gyro-multiplier with a self-exciting LF section (gyro-BWO with the normalized length $\hat{L} = 2.2$), and axial distribution of harmonics of the electron current.

Such a situation is quite typical for all types of auto-oscillators based on inertial electron

bunching, and it complicates the realization of the two-cavity scheme of the gyro-multiplier with a self-exciting LF section. Indeed, the HF generator is placed after the LF generator, i.e. in a region where the desired high harmonic electron current is small due to over-bunching, whereas the optimal place for the HF cavity should be somewhere in the middle of the LF cavity. The optimal place for the HF generation can be removed out of the LF generator, if the LF generator operates in a regime close to the small-signal one (when the electron current is close to the starting value). However, such a regime is quite difficult in the experimental realization for pulse operation.

One should notice that the use of a fixed frequency LF oscillator (like a gyrotron) leads to the serious additional problem of frequency synchronization between the two (LF and HF) oscillators. In other words, eigenfrequencies of these two oscillators must be divisible, $\omega_{HF} \approx n\omega_{LF}$, with quite a high level of accuracy,

$$\left|\frac{n\omega_{LF} - \omega_{HF}}{n\omega_{LF}}\right| < \frac{1}{Q_{LF}} + \frac{1}{Q_{HF}}, \qquad (17)$$

where $Q_{\mathrm{LF,\ HF}}$ are the Q-factors of cavities of LF and HF oscillators. This problem is even more complicated by the fact that the condition of frequency synchronization should be provided for the "hot" eigenfrequencies, so that the impact of the electron beam on the resonant eigenfrequencies of both oscillators must be taken into account.

3. Single-cavity homogeneous schemes

3.1 "Gyrotron-gyrotron" two-wave oscillator

As is seen from Fig. 3, the self-exciting LF section of a multiplier contains internally the optimal conditions for HF wave excitation. Therefore, a natural way to realize a self-exciting multiplier is to embed the HF oscillator into the LF structure, or, to put it another way, to combine both oscillators inside a single cavity.

This can be achieved in a cavity having eigenmodes with exactly divisible frequencies, so that both resonance conditions (1) and (2) will be fulfilled simultaneously (Fig. 4). In this scheme the maximum of the *n*-th current harmonic is placed in the region of HF resonance and more efficient coupling to the HF wave can be expected. One of the most convenient forms for such cavities is a piece of circular waveguide. In this case, rules (3) should be supplemented by the following requirement for the azimuthal indexes of the LF and HF modes:

$$m_{HF} = nm_{LF} \qquad (18)$$

To mitigate the problems of mode selection and sensitivity to the spread in electron velocities, the preferred type of LF and HF resonances are those of the gyrotron. Thus, one has to find a pair of $\mathrm{TE_{m,p}}$ and $\mathrm{TE_{(nm),s}}$ modes with divisible cut-off frequencies, $\omega_{(nm),s} = n\omega_{m,p}$. In general, the cut-off frequencies of one circular waveguide mode are not integral multiples of the other. For instance, to double the frequency of the $\mathrm{TE_{1,1}}$ mode with an axis-encircling electron beam (Fig. 5a), one can't find a near cut-off resonance with the $\mathrm{TE_{2,s}}$ modes. A similar situation takes place for other radial indexes *p* of $\mathrm{TE_{1,p}}$mode.

Nevertheless, for certain multiplication factors there exists a series of modes of a circular waveguide with a pair of divisible azimuthal indexes and quasi-divisible frequencies. Actually, the cut-off frequencies of a circular waveguide with radius *R*are proportional to either zeros of the corresponding Bessel functions (for TM modes), or their first derivatives, (for TE modes):

$$\omega_{m,p}^{TM} = cj_{m,p} / R, \quad \omega_{m,p}^{TE} = cj'_{m,p} / R \tag{19}$$

Here the azimuthal index, *m*, coincides with the order of the Bessel function and the radial index, *p*, coincides with the root number. Using McMahon's expansions for large zeros (*p*>>*m*) of Bessel functions [28], gives

$$\begin{aligned} j_{m,p} &= (p + m/2 - 1/4)\pi + O(m^2 / p) \\ j'_{m,p} &= (p + m/2 - 3/4)\pi + O(m^2 / p) \end{aligned} \tag{20}$$

According to (20), for a circular waveguide the following asymptotic relation exists:

$$5 j'_{m,p} \approx j'_{(5m),(5p-3)} \tag{21}$$

This means that for a frequency multiplying factor *n*=5 one can find "proper" pairs of TE modes, namely, $TE_{m,p}$ for the LF wave and $TE_{(5m),(5p-3)}$ for the HF wave. As an example of such a "proper" pair of modes, Fig. 5b shows the dispersion diagram for the case of the $TE_{1,3}$ and $TE_{5,12}$ modes.

In principle, an additional "proper" pair of modes may be also found, if one uses excitation of a TM wave at a multiplied frequency. Since

$$3 j'_{m,p} \approx j'_{(3m),(3p-2)} \tag{22}$$

there is a possibility to provide a frequency multiplication factor of n=3 choosing the following pair of modes: the LF $TE_{m,p}$ mode at the fundamental cyclotron resonance and the HF $TM_{(3m),(3p-2)}$ mode at the third harmonic. One should mention here that the excitation of a TM mode is possible and quite effective (in the case of relativistic electron energies) due to the interaction of particles with the axial component of the electric field of this mode [29].

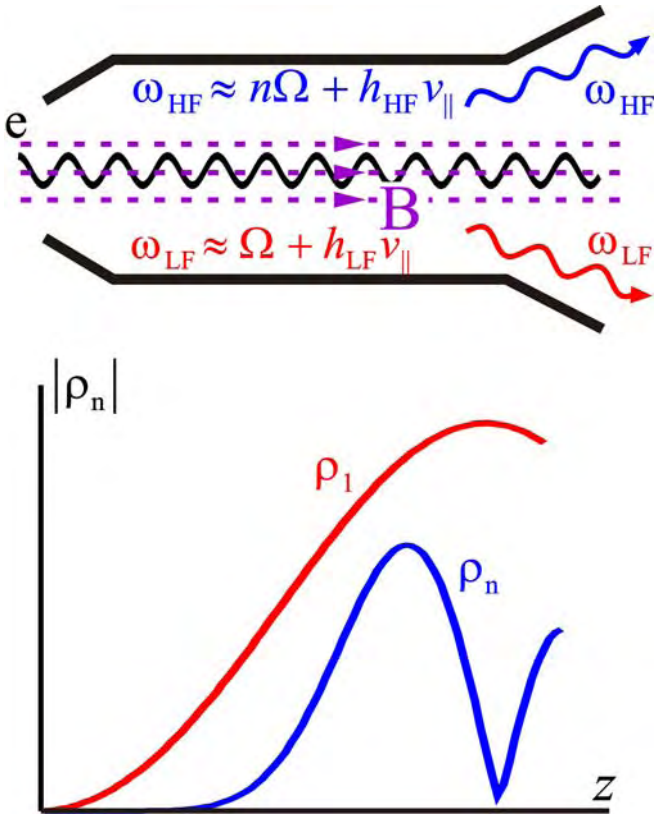

Fig. 4 Schematic of a single-cavity self-exciting gyro-multiplier, and typical axial distribution of harmonics of the electron current

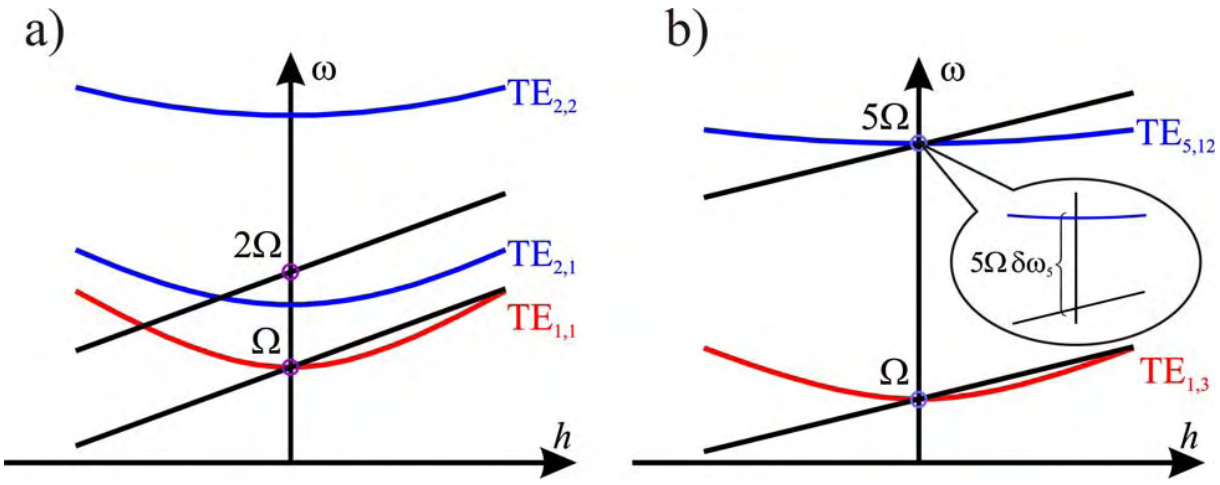

Fig. 5 Dispersion curves for frequency doubling (a) and frequency quintupling (b) in the case of a cavity with the circular cross-section.

Table 1 illustrates the relative difference in waveguide eigenfrequencies of the "proper" pairs of modes in the cases of frequency multiplication factors n =5 and n =3,

$$\delta\omega_5 = \left[5\omega_{1,p}^{TE} - \omega_{5,(5p-3)}^{TE}\right] / 5\omega_{1,p}^{TE}$$

$$\delta\omega_3 = \left[3\omega_{1,p}^{TE} - \omega_{3,(3p-2)}^{TM}\right] / 3\omega_{1,p}^{TE} \qquad (23)$$

p	$\delta\omega_5$	$\delta\omega_3$
1	- 14.3 %	- 15.5 %
2	- 1.3 %	- 1.4 %
3	- 0.50 %	- 0.54 %
4	- 0.27 %	- 0.29 %

Table 1. Relative frequency discrepancies, $\delta\omega_5$ and $\delta\omega_3$, in the cases of co-generation of $TE_{m,p}$ $TE_{(5m),(5p-3)}$ modes and $TE_{m,p}$ -$TM_{(3m),(3p-2)}$ modes, respectively, versus the radial index of the fundamental-harmonic mode.

Minimization of the frequency discrepancy, δω, can be achieved by shortening the cavity. Actually, the "cold" eigenfrequency of a near-cutoff mode of a cavity differs from the cutoff frequency:

$$\omega_{m,p,q} \approx \sqrt{\omega_{m,p}^{cutoff} + (q\pi c / L)^2} \qquad (24)$$

where $\omega_{m,p}^{cutoff}$ is the cutoff frequency determined by Eqs. (19), q is the axial index, and L is the cavity length. In the case of the "gyrotron-gyrotron" co-generation scheme, when both LF and HF waves are near-cutoff waves (q =1 for both of the waves), they have the same axial wavenumbers, $h = \pi / L$, whereas their cutoff frequencies differs almost by n times:

$$\omega_{HF}^{cutoff} \approx n\omega_{LF}^{cutoff} \qquad (25)$$

In this case, the frequency discrepancy is determined as follows:

$$n\omega_{LF} - \omega_{HF} \approx n\omega_{LF}^{cutoff} - \omega_{HF}^{cutoff} + \frac{(ch)^2(n^2-1)}{2\omega_{HF}^{cutoff}} \qquad (26)$$

Decreasing the cavity length leads to an increase of the axial wavenumber, h . This helps to compensate the negative (according to Table 1) discrepancy in the cutoff frequencies of the two modes, $n\omega_{LF}^{cutoff} - \omega_{HF}^{cutoff} < 0$. It is important to note, that manufacturing errors may change the absolute values of the eigenfrequencies, but the relative separation between frequencies is bound to the cavity geometry and thus is more stable.

Finally, the exact minimization of the frequency discrepancy, $n\omega_{LF} - \omega_{HF}$, can be achieved by using the effect of the electron beam on the eigenfrequency of the LF oscillator. Due to this fact, the "hot" LF eigenfrequency can be slightly tuned by changing either the operating magnetic field or the electron pitch factor.

As an example of the "gyrotron-gyrotron" scheme of a two-wave gyro-oscillator with frequency multiplication, a moderately relativistic device with parameters of the Large-Orbit Gyrotron experiment [30] was numerically studied. Table 2 shows parameters and results of simulation for this "gyrotron-gyrotron" scheme with the operating cogenerated $TE_{1,3}$ and

$TE_{5,12}$ modes.

Electron beam	axis-encircling 250 kV / 3 A pitch factor – 1 spread in transverse velocity – $\Delta\beta_\perp/\beta_\perp$=60%
Multiplication factor, *n*	5
LF / HF modes	$TE_{1,3,1}$ / $TE_{5,12,1}$
HF / LF wavelengths, λ_L / λ_H	2.5 mm / 0.5 mm
Cavity radius, *R*	3.5 mm
Magnetic field, *B*	6.0 T - 6.1 T
Magnetic field band	0.3%
LF efficiency	3%
HF efficiency	0.1%

Table 2 Parameters and results of simulation for the moderately-relativistic large-orbit “gyrotron-gyrotron” two-wave oscillator.

The most serious problem arising while designing the gyrotron cavity for a multiplier scheme is the necessity to shorten the cavity in order to minimize the frequency discrepancy (Fig. 6). Such shortening leads to a decrease in the diffraction Q-factor of the open gyrotron-type cavity. Since the experimental setup has a rather stringent limitation of the beam current (3 A), this leads to the problem of satisfying the starting conditions for the LF wave. This problem can be solved by optimization of the shape of the cavity wall. With an extended tapered input cutoff narrowing of the cavity (Fig. 6 b), the same cavity provides different characteristic axial lengths for the LF and HF near-cutoff waves (Fig. 6 c). Due to this fact, it is possible to provide good synchronization of the “cold” frequencies of the two waves whilst retaining a reasonably long cavity. The “hot” eigenfrequency of the LF wave is shown schematically in the spectrum (Fig. 6 b) in green. It is slightly shifted from the “cold” LF-wave eigenfrequency, and it can be tuned by a change of the operating magnetic field, so that the exact frequency synchronization can be satisfied.

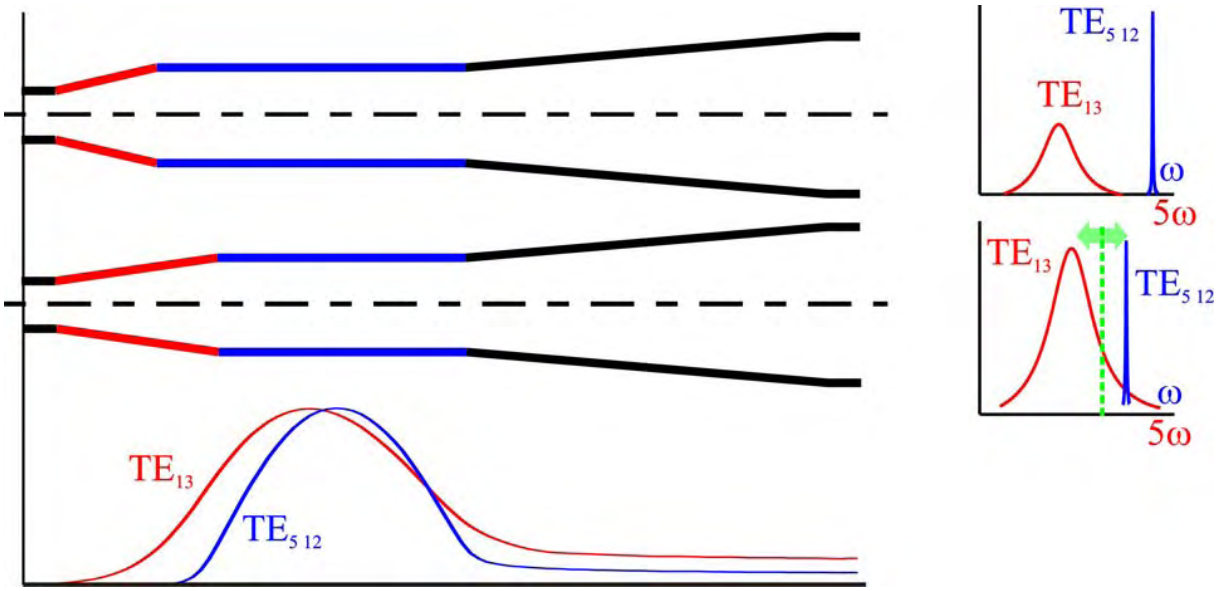

Fig.6 The "gyrotron-gyrotron" two-wave oscillator. Shape of the cavity and "cold" eigenmode spectra for the non-optimized (a) and optimized (b) cavities, and axial structures of the "cold" eigenmodes in the optimized case (c).

Since the magnetic field is responsible for the frequency synchronization in this system, the output power of the HF wave is very sensitive to its value. According to simulations (Figs. 7 and 8), the output HF power in this system can be as high as 700 W; however, it is provided in a very narrow magnetic field band (0.3%), whereas the magnetic field band of the LF gyrotron amounts to a few percent. This fact is illustrated in Fig. 8, which shows LF-wave starting current and a region of high-power ($P_{HF,out}$ >100 W) generation of the HF wave on the (magnetic field) – (electron current) plane. One should notice that this problem is not a critical issue for permanent or cryo-magnets, but it may represent a certain difficulty in realization of pulsed systems.

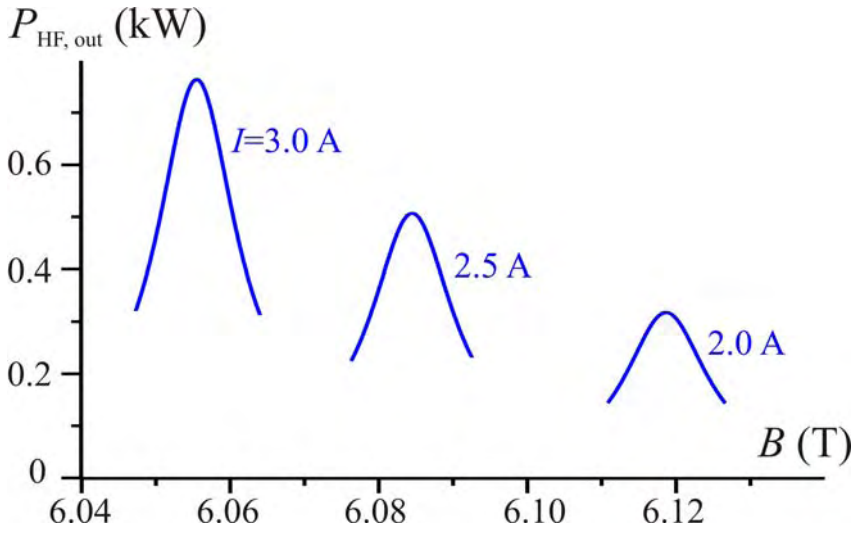

Fig.7 The "gyrotron-gyrotron" two-wave oscillator. Output power of the HF wave versus the magnetic field at various values of the electron current.

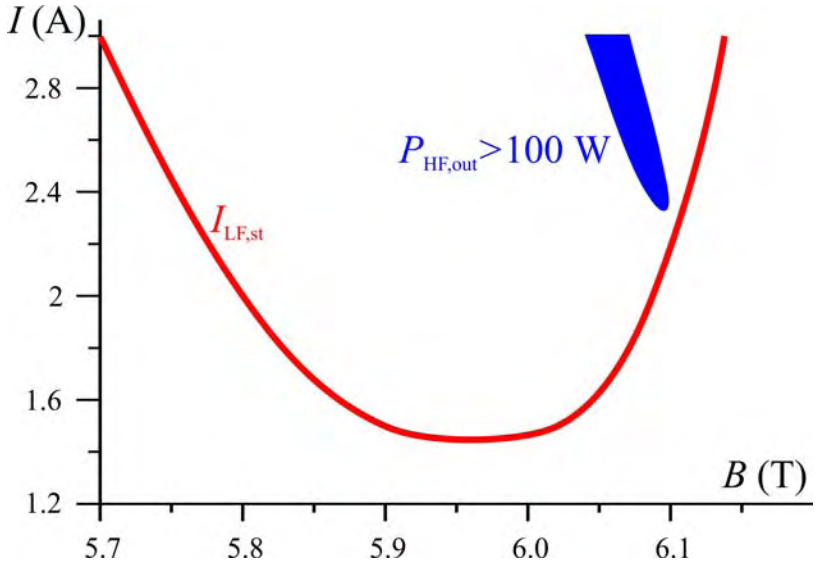

Fig.8 The "gyrotron-gyrotron" two-wave oscillator. Starting current of the LF wave (red) and region of high-power generation of the HF wave (blue) on the "(magnetic field) – (electron current)" plane.

Another serious (but solvable) problem of the "gyrotron-gyrotron" scheme is the necessity of a special microwave system which should provide separation of the output radiation of two spectral components having the same group velocities. This problem is complicated by a great difference in the power of these two modes. In the example discussed above, there arises the problem of separating the HF wave with a power of ~10^2W from the LF wave with a power of ~10^4W.

3.2 "Gyrotron-TWT" two-wave oscillator

As mentioned in Sect. II, in the case of a homogeneous magnetic field the co-generation of two waves is possible, if their frequencies, transverse wavenumbers and axial wavenumbers are divisible {see Eqs. (1-4)}. However, such a condition for the axial wavenumbers of the two co-generating waves is only approximate in character {Eq. (4)}. This fact could be used to provide co-generation of two modes of different types, namely, near-cutoff (gyrotron-type) LF wave and a traveling (TWT-type) HF wave. Actually, in the case of the gyrotron-type LF wave, resonance condition (1) can be re-written in the following form:

$$\omega_{LF} = \Omega + \delta \tag{27}$$

Here δ is the mismatch of the resonance for this wave. According to the gyrotron theory [2], the characteristic value of this mismatch is determined by the length of the oscillator,

$$\delta = \omega_{LF} - \Omega \sim 2\pi v_{\|} / L \tag{28}$$

As for the TWT-type HF wave, it should be close to exact cyclotron resonance,

$$\omega_{HF} = n\omega_{LF} \approx n\Omega + h_{HF} v_{\|} \tag{29}$$

Thus, according to Eqs. (27) - (29), the axial wavenumber of the traveling HF wave can be large enough in the case of a short cavity,

$$h_{\mathrm{HF}} \sim n\delta / \mathrm{v}_{\|} \sim n2\pi / L \tag{30}$$

One should take into account that the axial wavenumber of the near-cutoff mode is also determined by the cavity length, $h_{\mathrm{LF}} \approx \pi / L$. Therefore, the ratio of group velocities of the two waves is estimated as follows:

$$V_{gr}^{HF} \Big/ V_{gr}^{LF} = \frac{h_{HF}\omega_{LF}}{h_{LF}\omega_{HF}} \approx 2 \tag{31}$$

Evidently, similar to the previous "gyrotron-gyrotron" case, the co-generation of a gyrotron LF wave and an "almost gyrotron" HF wave (i.e. a traveling wave with a small group velocity) can be provided, if the cut-off frequencies of the waves are almost divisible, cutoff cutoff:

$$\omega_{HF}^{cutoff} \approx n\omega_{LF}^{cutoff} \tag{32}$$

However, unlike the case of n =5, the HF-wave cutoff frequency should be slightly lower,

cutoff cutoff than the multiplied cutoff frequency of the LF-wave, $\omega_{HF}^{cutoff} < n\omega_{LF}^{cutoff}$. This situation is realized in the "neighboring" case of n =6 (Fig. 9).

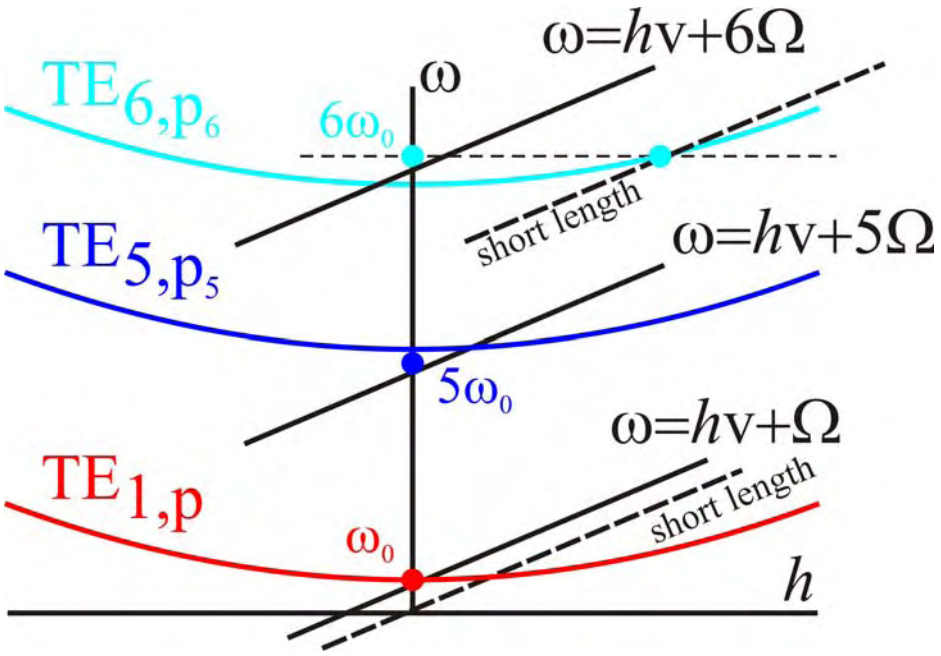

Fig. 9 Dispersion diagram of "proper" pairs of TE modes for the case of multiplication factors n =5 and n =6. Oblique lines illustrate the electron cyclotron dispersion characteristic for the cases of long (solid lines) and short (dashed lines) operating cavity.

The main peculiarity of the "gyrotron-TWT" scheme is significantly different group velocities of the two co-generating waves. This allows the use of an operating cavity with a small cutoff narrowing at its output. Such narrowing closes the near-cutoff LF wave inside the cavity, whereas the traveling HF wave is carried out from the cavity. Part of the power of this wave can be reflected from the output narrowing, so that a feedback for the HF oscillator would be provided.

The use of a cavity closed for the near-cutoff LF wave provides a number of advantages. First of all, the problem of mode separation is solved automatically. Secondly, the high Q-factor of the LF wave readily gives satisfaction of the starting conditions for this wave even in a short cavity, which helps to provide frequency synchronization of the two waves. In addition, according to Eq. (28), in the case of a short cavity the excitation of the LF oscillator occurs when the cyclotron frequency is significantly lower than the frequency of the near-cutoff LF wave. This fact helps to provide cyclotron resonance with the traveling HF wave (Fig.9).

Fig.10 illustrates results of simulation of a moderately-relativistic large-orbit "gyrotron-TWT" two-wave oscillator with parameters analogous to the "gyrotron-gyrotron" variant (see Table 2). In this oscillator, the gyrotron LF $TE_{1,4,1}$ wave is excited at the fundamental cyclotron resonance by a 250 kV/3A axis-encircling electron beam at a wavelength of 2.5 mm, whereas the HF $TE_{6,20}$ wave is excited at the sixth cyclotron harmonic at a wavelength of 0.42 mm. The difference in the group velocities of these two waves (V_{gr} =0.08c for the LF wave and V_{gr} =0.15c for the LF wave) was consistent with Eq. (31). Such a significant difference ensured the cavity is closed for the LF wave.

It has been assumed in these simulations, that the HF-wave reflection from the output narrowing of the cavity is absent. In this situation, the output power of the HF wave is about 100 W, which is significantly lower compared to the "gyrotron-gyrotron" variant (700 W). At the same time, the magnetic field band for the effective co-generation of the two modes amounts to a few percent, and is significantly broader than the "gyrotron-gyrotron" magnetic field band (0.3%).

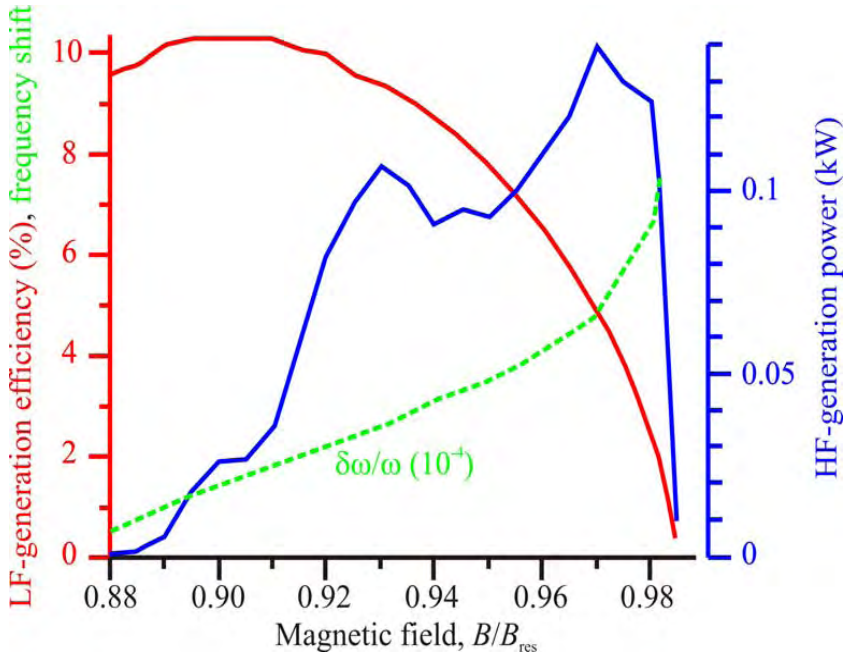

Fig.10 The "gyrotron-TWT" two-wave oscillator. Efficiency of the LF wave generation (red), HF-wave output power(blue), and the "hot" shift of the eigenfrequency (green) versus the magnetic field (normalized to its value correspondingto the electron cyclotron resonance with the LF wave).

It is interesting, that a change of the magnetic field leads to a small change of the "hot" eigenfrequency of this system (the green curve in Fig. 10). This change amounts to $\delta\omega/\omega \sim 10^{-3}$, and is of the order of the frequency band of the LF oscillator determined by the Ohmic

quality of the near-cutoff LF wave $\delta\omega/\omega \sim Q_{LF}^{-1}$ (since this mode is supposed to be closed inside the cavity, its diffraction quality is infinitely high). Thus, the "gyrotron-TWT" variant of the two-wave oscillator allows a narrowband range of frequency tuning.

Certainly, since the group velocity of the HF wave is quite small, the narrowing at the cavity output will provide reflection of part of the power of this wave and, therefore, some feedback for the HF oscillator. According to simulations with a power reflection coefficient for this wave of R =20% , the output HF-wave power reaches 300 W, whereas in the case of R =50% it becomes as high as in the "gyrotron-gyrotron" scheme (650 W).

At the same time, the increase of the Q-factor of the HF wave results in narrowing of the frequency band of the HF eigenmode, and, therefore, in narrowing the magnetic field tuning band for the effective HF-wave generation.

4. Single-cavity inhomogeneous schemes

As shown in the previous Sections, in the case of a uniform magnetic field, the properties of the eigenmodes of a circular waveguide allow the realization of either a "gyrotron-gyrotron" scheme of the two-wave oscillator with a frequency multiplication factor of n =5, or the "neighboring" case of the "gyrotron – TWT (almost gyrotron)" scheme with n =6. If one needs to realize co-generation of two waves with significantly different group velocities and with a different frequency multiplication factor, one should use magnetic field profiling. A possible variant of such a profiled two-wave oscillator is shown in Fig. 11. Its interaction region represents three sections differing from each other by the value of the magnetic field. Inside the first and the third sections, the magnetic field is close to resonance with the near-cutoff gyrotron-type LF wave. In the middle section, where the HF harmonic current, ρ_n , reaches its maximum, the magnetic field is close to resonance with a traveling HF wave, which can be either a forward propagating (TWTtype) or a backward-propagating (BWO-type).

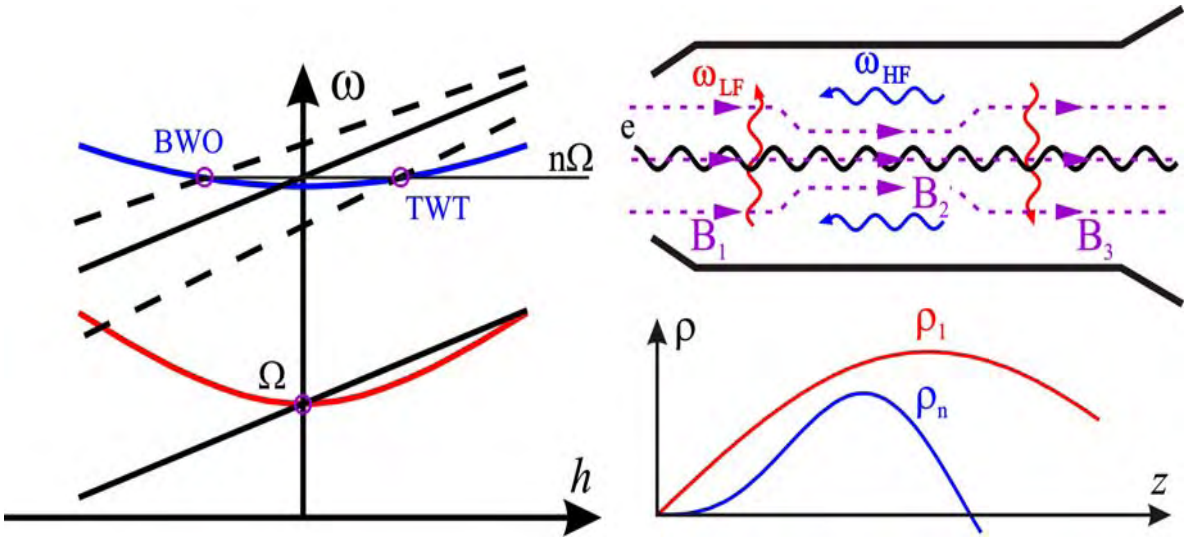

Fig. 11 Two-wave oscillator with profiled magnetic field.

From the point of view of the LF oscillator, this scheme is analogous to a klystron with positive feedback. Actually, the first section operates similar to the modulator, whereas the third one is similar to the output resonator. The "drift region" of this "klystron" is used to

produce HF emission.

It is important that the magnetic field profiling slightly disrupts the process of electron bunching. Actually the change of the magnetic field leads to conversion between the transverse and longitudinal components of the electron momentum. This causes a change in the parameters of electron-wave coupling and of inertial bunching and thus affects the bunching speed. As for the coupling parameter, it is proportional to the transverse electron momentum and grows with increasing magnetic field. To understand the behavior of the inertial bunching factor one should start from the non-asymptotic equation for electron phase for the LF wave:

$$\frac{d\theta}{dz} = \frac{\omega_{LF} - \Omega}{\mathrm{v}\|} \tag{33}$$

Let us consider the case of a precise gyrotron-type resonance in the modulating section,

$$\omega_{LF} = \Omega_1 = \hat{\Omega}_1 / \gamma_0 \tag{34}$$

where $\hat{\Omega}_1 = eB_1 / mc$ is the non-relativistic cyclotron frequency in the first section, and γ_0 is the initial electron Lorentz-factor. In the case of the gyrotron-type electron-wave interaction, the axial momentum of the electrons, $p_\| = \mathrm{m}\gamma\mathrm{v}_\|$, does not vary in the first section. Therefore, the modulation of the transverse electron momentum in the first section, $p_\perp$, is connected with the modulation of the relativistic electron Lorentz-factor in the following way:

$$p_{\perp 0}\delta p_\perp = m^2 c^2 \gamma_0 \delta\gamma \tag{35}$$

In the case of a constant magnetic field, the axial momentum also remains constant, $\gamma\mathrm{v}_\| = \mathrm{const}$, and electron bunching in the drift region is described by the following equation:

$$\frac{d\theta}{dz} = \frac{\gamma\omega_{LF} - \hat{\Omega}_1}{\gamma \mathrm{v}_\|} = \frac{\hat{\Omega}_1}{\gamma_0^2 \mathrm{v}_{\|0}} \delta\gamma \tag{36}$$

However, in the case of a profiled magnetic field, in the region of transition from the first section to the second, the transverse electron momentum changes according to the adiabatic invariant, $p_\perp^2 / B = \mathrm{const}$. One can assume that in the second section the interaction between the electrons and the radiation is absent, as this wave is far from the resonance. This means that the total electron momentum is constant, so that the transition from the first section to the second one is described as follows:

$$p_{\|0}^2 + (p_{\perp 0} + \delta p_\perp)^2 = (p_{\|2} + \delta p_\|)^2 + (p_{\perp 0} + \delta p_\perp)^2 \frac{B_2}{B_1} \tag{37}$$

Here $p_{\|2}$ is the non-modulated part of the axial electron momentum in the second section, and $\delta p_\|$ is the modulation of the axial momentum caused by the transformation of the modulation of the transverse momentum. According to (37), this modulation is determined as follows:

$$p_{\|2}\delta p_{\|} = -\frac{\Delta B}{B_1} p_{\perp 0}\delta p_{\perp} = -\frac{\Delta B}{B_1} m^2 c^2 \gamma_0 \delta\gamma \qquad (38)$$

where $\Delta B = B2 - B1$. Taking into account this modulation in Eq. (33), instead of Eq. (36) one obtains the following equation describing the electron bunching in the drift region:

$$\frac{d\theta}{dz} = \frac{\gamma\omega_{LF} - \hat{\Omega}_1}{(p_{\|2} + \delta p_{\|})/m} = \frac{\hat{\Omega}_1}{\gamma_0^2 \mathrm{v}_{\|0}} \left[1 - \left(\frac{c}{\mathrm{v}_{\|2}} \frac{\Delta B}{B_1} \right)^2 \right] \delta\gamma \qquad (39)$$

According to Eq. (15), a large change in magnetic field,

$$\left| \Delta B / B \right| \sim \mathrm{v}_2 / c \qquad (40)$$

can noticeably modify the process of electron bunching in the second section and is best avoided. In other words, similar to the "gyrotron-gyrotron" scheme, the frequency discrepancy here is an object of optimization.

We have studied a moderately-relativistic large-orbit "gyrotron-TWT" two-wave oscillator with profiled magnetic field with parameters, which are similar to parameters of the uniform devices ("gyrotron-gyrotron" and "gyrotron-TWT") considered in the previous section (Table 3). In this "gyrotron-TWT" oscillator, the gyrotron LF $TE_{1,4,1}$ wave is excited at the fundamental cyclotron resonance by a 250kV/3A axis-encircling electron beam at a wavelength of 2.5 mm, whereas the HF $TE_{5,16}$ wave is excited at the fifth cyclotron harmonic at a wavelength of 0.5 mm. In contrast to the uniform "gyrotron-TWT" (Sect. III B), the traveling HF wave has quite a high group velocity ($V_{gr} \approx 0.3c$). This leads to a stronger sensitivity to the velocity spread. At a moderate spread ($\delta \mathrm{v}_{\perp} / \mathrm{v}_{\perp} = 20\%$), the output HF power of this device (~100W), as well as the magnetic field band (2%) are similar to the uniform "gyrotron-TWT" (Sect. III B).

Electron beam	cylindrical, axis-encircling 250 kV / 3 A pitch factor ~ 1 spread in perpendicular velocity $-\Delta\beta_{\perp}/\beta_{\perp}=20\%$
Multiplication factor	5
LF / HF modes	$TE_{1,4,1}$ / $TE_{5,16}$
HF / LF wavelengths	2.5 mm / 0.5 mm
Magnetic field band	2%
LF efficiency	10%
HF efficiency	0.015%

Table 3. Parameters and results of simulation for the moderately-relativistic large-orbit "gyrotron-TWT" two-wave oscillator with profiled magnetic field.

Figure 12 illustrates the electron-wave interaction in the "gyrotron-TWT" two-wave oscillator with a profiled magnetic field. It is shown that the interaction with different waves

is split into different parts of the operating cavity. Actually, the HF-wave excitation occurs in the middle part of the interaction region, where the magnetic field is close to the value corresponding to the high-harmonic cyclotron resonance with this wave. The increase of the LF efficiency occurs in the output part of the cavity; fast oscillations of the LF efficiency in the middle part correspond to oscillation of the energy of the electron bunch in the field of a non-resonant wave.

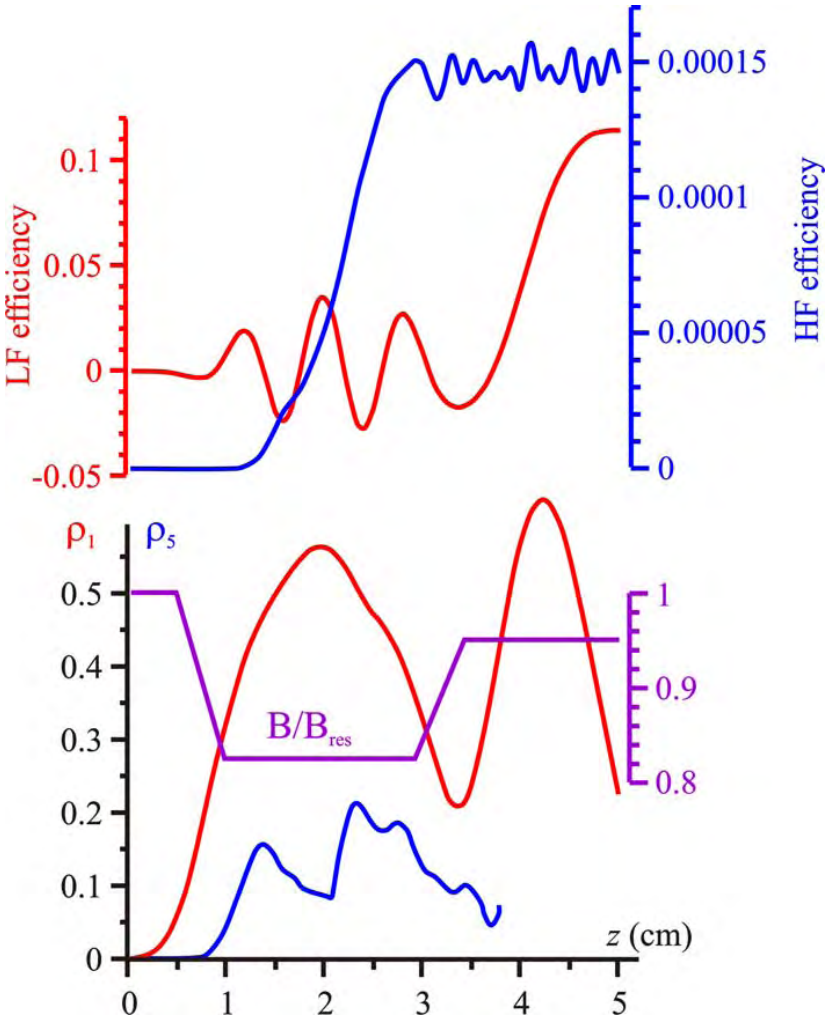

Fig. 12 The "gyrotron-TWT" two-wave oscillator with profiled magnetic field. Efficiency of the LF wave generation and electron-current harmonic at the LF-wave frequency (red curves), similar plots for the HF wave (blue curves), and the magnetic field profile (violet curve).

In the example described the factor of frequency multiplication equals n =5. We have chosen such a value advisedly in order to simplify the comparison between this scheme and ones described in the previous Sections. As a matter of fact, the scheme with a profiled magnetic field allows any other multiplication factor, such as n =4 or n =3. The use of smaller factors may be attractive for achieving higher radiation efficiency at the expense of a higher magnetic field.

5. Conclusion

An attractive scheme of a self-exciting gyro-multiplier is a single-cavity two-wave oscillator, where the high-frequency, high-harmonic generator is embedded into the

low-frequency fundamental-harmonic resonator. In the case of a uniform magnetic field, it is possible to realize a limited number of variants, including the "gyrotron-gyrotron" multiplier (with multiplication factor n=5) and the "gyrotron-TWT" multiplier (n=6). Simulations demonstrate the possibility of gyro-oscillators in the THz frequency range with output powers of over 100 W and narrow-band frequency tuning at moderately-relativistic (250 keV) electron energies.

The use of a special profile of the magnetic field can provide a fundamental-harmonic oscillator with a klystron-like electron-wave interaction. The middle part of this "klystron" can operate as a high-harmonic generator. In this scheme, any types of co-generating waves (standing, forward, backward) and any multiplication factors are possible.

Acknowledgments

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