Invited Paper

Channel estimation for intelligent reflecting surface aided multi-user MISO terahertz system

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Abstract: Intelligent reflecting surface (IRS) is considered as a promising application in terahertz (THz) communications since it is able to enhance the THz communication with no additional power consumptions. In this letter, we consider the channel estimation problem for an IRS-aided THz multi-user multi-input single-output (MISO) system with lens antenna array. The main challenge of the problem is that we need to estimate multiple channels and some of the channels are cascaded. To deal with the problem, we propose a two-stage channel estimation scheme, where we set different IRS modes to estimate different channels for each stage. In stage 1, we set the IRS to an absorbing mode and estimate the channel without IRS. Removing the influence of the prior estimated channel, in stage 2, we estimate the channel with IRS by setting the IRS to a perfect reflecting mode. And we decompose the total channel estimation problem into a series of independent problems, where we estimate each independent channel component with a least square method.

Keywords: Intelligent reflecting surface (IRS), Terahertz (THz) communication, channel estimation

doi:

1. Introduction

Intelligent reflecting surface (IRS), as a promising technique for future wireless systems, such as terahertz (THz) communications, has attracted growing research interest in both academia and industry over recent years [1, 2]. An IRS is a physical meta-surface consisting of a large number of reflecting elements, where each element is equipped with a simple low-cost sensor [3]. And each element is able to reflect incident electromagnetic waves independently by adjusting its phase-shift. Compared to traditional relay schemes that enhance source-destination transmission by generating new signals, IRS does not buffer or process any incoming signals but only reflects the wireless signal as a passive planar array, which incurs no additional power consumptions [4, 5].

Previous works about the IRS are mainly focused on optimizing secrecy-rate and data-rate by designing the phase-shifts of the IRS while assuming perfect channel state information (CSI) is obtained by both the base station (BS) and the IRS [6-10]. However, it is difficult to obtain the perfect CSI since the IRS cannot process any induced signals or emit any pilot signals. Therefore, the BS needs to estimate all the channels between the BS and the user, which includes the channel between the (BS, IRS), (IRS, user), and (BS, user). To the best of our knowledge, there are limited literatures considering the channel estimation problem for the IRS-aided system.

Thus, in this letter, we investigate the channel estimation problem for the IRS-aided THz multi-user multi-input single output (MISO) system with lens antenna array. To solve the problem, we propose a two-stage channel estimation scheme, where we set different IRS modes for the channel estimation in different stages. In stage 1, we estimate the channel between the BS and the user by setting the IRS to an absorbing mode which is able to absorb all induced signals by the IRS. Removing the influence of the prior estimated channel, in stage 2, we set the IRS to a perfect reflecting mode, which can reflect all induced signals by the IRS with few losses. And we find that the channel with the IRS a cascaded channel. To estimate it, we decompose the total channel estimation problem into a series of independent problems, where we estimate each channel component with a least square method.

2. System model and problem formulation

2.1 System model

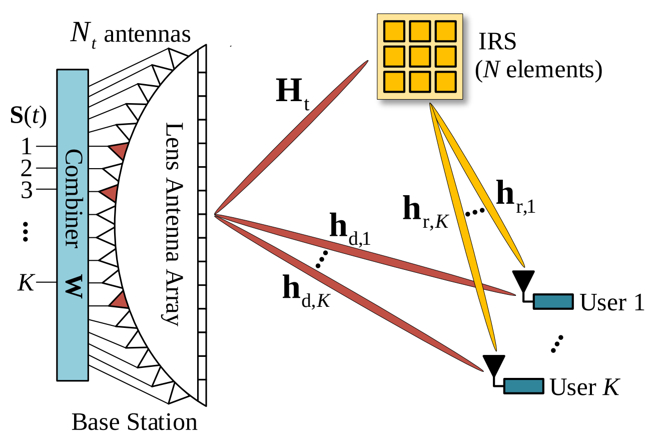

Fig. 1 IRS-aided THz multi-user MISO system with lens antenna array

As shown in Fig. 1, we consider an uplink THz multiuser MISO system, where a BS, which consists of a one dimensional lens antenna array with N_t elements, simultaneously receives signals from K single-antenna users. To enhance the THz communication, an IRS equipped with N passive elements is installed on a surrounding wall to overcome unfavorable propagation conditions and enrich the channel with more paths. For each path, due to the severe propagation loss in the THz communication, we only consider a single reflection signal by the IRS and ignore other signals reflected by the IRS more than one time. And we assume only one data stream needs to be transmitted by each user. In t th instant, each user sends a pilot signal, denoted by $\{s_k(t)\}_{k=1}^{K} \in \mathcal{C}$, to the BS over two ways. One way is achieved by the directly channel between the BS and the user. Another way is achieved by the IRS. The IRS can reflect THz signals to the BS by a diagonal phase-shift matrix $\mathbf{\Theta} \in \mathcal{C}^{N\times N}$ which will be discussed later. Therefore, the received signal $\mathbf{y}(t) = \mathcal{C}^{N_t\times 1}$ at the BS can be expressed as

$$\mathbf{y}(t) = (\mathbf{H}_{\mathrm{t}}\mathbf{\Theta}\mathbf{H}_{\mathrm{r}} + \mathbf{H}_{\mathrm{d}})\mathbf{s}(t) + \mathbf{n}(t), \tag{1}$$

where $\mathbf{s}(t) = [s_1(t), s_2(t), \cdots, s_K(t)]^T \in \mathcal{C}^{K\times 1}$ is the pilot vector for the channel estimation process, $\mathbf{H}_{\mathrm{r}} = [\mathbf{h}_{\mathrm{r},1}, \mathbf{h}_{\mathrm{r},2}, \cdots \mathbf{h}_{\mathrm{r},K}] \in \mathcal{C}^{N\times K}$ (resp. $\mathbf{H}_{\mathrm{t}} \in \mathcal{C}^{N_t\times N}$) is the channel between the IRS and the user (resp. between the BS and the IRS), $\mathbf{H}_{\mathrm{d}} = [\mathbf{h}_{\mathrm{d},1}, \mathbf{h}_{\mathrm{d},2}, \cdots \mathbf{h}_{\mathrm{d},K}] \in \mathcal{C}^{N_t\times K}$ is the channel between the BS and the user, and $\mathbf{n}(t) \in \mathcal{C}^{N_t\times 1}$ is zero-mean additive white Gaussian noise where we denote δ^2 as the noise power. In addition, the phase-shift matrix can be represented as $\mathbf{\Theta} = \mathrm{diag}\left(\left[\beta e^{j\theta_1}, \beta e^{j\theta_2}, \cdots, \beta e^{j\theta_N}\right]^T\right)$ where $\{\theta_n\}_{n=1}^{N} \in [0, 2\pi]$ represents the phase shift for the n th reflecting element, and $\beta \in [0,1]$ is an amplitude reflection coefficient on the incident signals. It is worth noting that when we set $\beta = 0$, the IRS mode turns to absorbing mode, which means that all signals get to the IRS will be absorbed. And when we set $\beta = 1$, the IRS mode turns to perfect reflecting mode, which means that all signals get to the IRS can be reflected with few losses. To estimate the channels $\mathbf{H}_{\mathrm{t}}$, $\{\mathbf{h}_{\mathrm{r},k}\}_{k=1}^{K}$, and $\{\mathbf{h}_{\mathrm{d},k}\}_{k=1}^{K}$, we use total T instants for the channel estimation. And we divide T into M blocks, where each block consists of K instants. Thus, we have $T = MK$. For the m th block ($m = 1, 2, \cdots, M$), the received signal $\mathbf{y}_m \in \mathcal{C}^{N_t\times K}$ at the BS can be written as

$$\mathbf{y}_m = (\mathbf{H}_{\mathrm{t}}\mathbf{\Theta}\mathbf{H}_{\mathrm{r}} + \mathbf{H}_{\mathrm{d}})\mathbf{s}_m + \mathbf{n}_m, \tag{2}$$

where $\mathbf{s}_m = [\mathbf{s}(mk - k + 1), \cdots, \mathbf{s}(mk)] \in \mathcal{C}^{K\times K}$ is the m th pilot matrix. To normalize the power of the pilot signal to unit, $\mathbf{s}_m$ satisfies $\mathrm{E}(\mathbf{s}_m\mathbf{s}_m^H) = \mathbf{I}_K$. And $\mathbf{n}_m \in \mathcal{C}^{N_t\times K}$ is the noise matrix.

In terms of $\mathbf{H}_{\mathrm{t}}$, $\{\mathbf{h}_{\mathrm{r},k}\}_{k=1}^{K}$, and $\{\mathbf{h}_{\mathrm{d},k}\}_{k=1}^{K}$, motivated by [11], we use a modified Saleh-Valenzuela model to capture the characteristics of the THz channel, which is comprised of several paths by reflection and directly transmission. Specifically, the channel response of $\mathbf{H}_{\mathrm{t}}$, $\{\mathbf{h}_{\mathrm{r},k}\}_{k=1}^{K}$, and $\{\mathbf{h}_{\mathrm{d},k}\}_{k=1}^{K}$ can be respectively given by

$$\begin{aligned}\mathbf{H}_{\mathrm{t}} &= \sqrt{\frac{N_{\mathrm{t}}\times N}{L_t}}\sum_{i=1}^{L_t}\alpha_i\mathbf{a}_t\left(\psi_i^t\right)\mathbf{a}_r^H\left(\psi_i^r\right),\\ \left\{\mathbf{h}_{\mathrm{r},k}\right\}_{k=1}^{K} &= \sqrt{\frac{N}{L_r}}\sum_{j=1}^{L_r}\alpha_j\mathbf{a}_t\left(\psi_j^t\right),\\ \left\{\mathbf{h}_{\mathrm{d},k}\right\}_{k=1}^{K} &= \sqrt{\frac{N_t}{L_d}}\sum_{l=1}^{L_d}\alpha_l\mathbf{a}_t\left(\psi_l^t\right),\end{aligned} \tag{3}$$

where L_t (resp. L_r and L_d) is the number of paths for channel $\mathbf{H}_{\mathrm{t}}$ (resp. $\left\{\mathbf{h}_{\mathrm{r},k}\right\}_{k=1}^{K}$ and $\left\{\mathbf{h}_{\mathrm{d},k}\right\}_{k=1}^{K}$), $\left\{\psi_i^t\right\}_{i=1}^{L}$ (resp. $\left\{\psi_i^r\right\}_{i=1}^{L}$) is the spatial direction, which can be defined as $\left\{\psi_i^t\right\}_{i=1}^{L} \triangleq \frac{d}{\lambda}\sin\left\{\varphi_i^t\right\}_{i=1}^{L}$ (resp. $\left\{\psi_i^r\right\}_{i=1}^{L} \triangleq \frac{d}{\lambda}\sin\left\{\varphi_i^r\right\}_{i=1}^{L}$), where $\left\{\varphi_i^t\right\}_{i=1}^{L}$ (resp. $\left\{\varphi_i^r\right\}_{i=1}^{L}$) is the physical direction, λ is the wavelength of carrier, and d is the antenna spacing or reflecting-element spacing. In addition, $\left\{\alpha_i\right\}_{i=1}^{L}$ is the complex gain for path i, which is mainly contributed by transmission losses and molecular absorbing losses in THz communication. And $\mathbf{a}(\psi)$ is the array steering vector. For a typical uniform linear array with $\bar{N}$ antennas, $\mathbf{a}(\psi)$ can be represented as $\mathbf{a}(\psi)=\left[1,e^{j2\pi\psi},\cdots,e^{j2\pi\psi\left(\bar{N}-1\right)}\right]^T\Big/\sqrt{\bar{N}}$.

Furthermore, the conventional channel (3) in the spatial domain can be transformed to the beamspace channel by employing the lens antenna array with a set of bases, which can be expressed as $\mathbf{U}=\left[\mathbf{a}(\bar{\psi}_1),\mathbf{a}(\bar{\psi}_2),\cdots,\mathbf{a}\left(\bar{\psi}_{N_t}\right)\right]^H$, where $\left\{\bar{\psi}_i\right\}_{i=1}^{N_t}\in\left[-0.5,0.5\right]$ denotes the spatial direction. And with the transformed signals in the beamspace domain, the BS can employ a combiner $\mathbf{W}_m\in\mathcal{C}^{K\times N_t}$ to combine the above signals. Then, for the mth block, the combined signal $\mathbf{R}_m\in\mathcal{C}^{K\times K}$ can be obtained as

$$\mathbf{R}_m=\mathbf{W}_m\mathbf{U}\left[\left(\mathbf{H}_{\mathrm{t}}\mathbf{\Theta}\mathbf{H}_{\mathrm{r}}+\mathbf{H}_{\mathrm{d}}\right)\mathbf{s}_m+\mathbf{n}_m\right] \tag{4}$$

2.2 Problem formulation

During the channel estimation, the estimated channel can be denoted as $\hat{\mathbf{H}}_{\mathrm{t}}$, $\hat{\mathbf{H}}_{\mathrm{r}}$, and $\hat{\mathbf{H}}_{\mathrm{d}}$. And the estimated combined signal $\hat{\mathbf{R}}_m$ is able to be represented as $\hat{\mathbf{R}}_m=\mathbf{W}_m\mathbf{U}\left(\hat{\mathbf{H}}_{\mathrm{t}}\mathbf{\Theta}\hat{\mathbf{H}}_{\mathrm{r}}+\hat{\mathbf{H}}_{\mathrm{d}}\right)\mathbf{s}_m$. Our interest lies in minimizing the Euclidean distance between $\hat{\mathbf{R}}_m$ and $\mathbf{R}_m$ by estimating the channel $\hat{\mathbf{H}}_{\mathrm{t}}$, $\hat{\mathbf{H}}_{\mathrm{r}}$, and $\hat{\mathbf{H}}_{\mathrm{d}}$, which is written as

$$\min_{\hat{\mathbf{H}}_{\mathrm{t}},\hat{\mathbf{H}}_{\mathrm{r}},\hat{\mathbf{H}}_{\mathrm{d}}} \sum_{m=1}^{M} \left\| \mathbf{R}_m - \hat{\mathbf{R}}_m \right\|_F^2 \tag{5}$$

3. Channel estimation scheme

In this section, we seek to solve problem (5) with estimating the channel $\mathbf{H}_{\mathrm{t}}$, $\{\mathbf{h}_{\mathrm{r},k}\}_{k=1}^{K}$, and $\{\mathbf{h}_{\mathrm{d},k}\}_{k=1}^{K}$. And we propose a two-stage channel estimation scheme, where we first estimate $\{\mathbf{h}_{\mathrm{d},k}\}_{k=1}^{K}$ by turning the IRS mode to the absorbing mode, and then we estimate $\{\mathbf{h}_{\mathrm{r},k}\}_{k=1}^{K}$ by removing the influence of $\{\mathbf{h}_{\mathrm{d},k}\}_{k=1}^{K}$. For the second-stage channel estimation, we decompose the total channel estimation problem into a series of independent problems, where we estimate each channel component with a least square method.

Specifically, we first multiply the know pilot matrix $\mathbf{s}_m^H$ on the right side of (4). Since we have $\mathbf{s}_m\mathbf{s}_m^H = \mathbf{I}_K$, the measurement matrix $\mathbf{Z}_m \in \mathcal{C}^{K\times K}$ can be obtained by

$$\mathbf{Z}_m = \mathbf{R}_m\mathbf{s}_m^H = \mathbf{W}_m\mathbf{U}\left[\left(\mathbf{H}_{\mathrm{t}}\boldsymbol{\Theta}\mathbf{H}_{\mathrm{r}} + \mathbf{H}_{\mathrm{d}}\right) + \mathbf{n}_m\mathbf{s}_m^H\right] \tag{6}$$

And each column of $\mathbf{Z}_m$, denoted by $\{\mathbf{Z}_m(:,k)\}_{k=1}^{K}$, is the measurement vector for the sub-channel of user k. After M block's measurement, we can obtain a $T\times 1$ measurement vector for user k, which can be written as

$$\begin{aligned}\tilde{\mathbf{Z}}_k &= \left[\mathbf{Z}_1^T(:,k), \mathbf{Z}_2^T(:,k), \cdots, \mathbf{Z}_M^T(:,k)\right]^T \\ &= \mathbf{W}\mathbf{U}\left(\mathbf{H}_{\mathrm{t}}\boldsymbol{\Theta}\mathbf{H}_{\mathrm{r},k} + \mathbf{H}_{\mathrm{d},k}\right) + \mathbf{N}(:,k)\end{aligned} \tag{7}$$

where vector $\mathbf{N}(:,k) = \left[\mathbf{N}_1^T(:,k), \mathbf{N}_2^T(:,k), \cdots, \mathbf{N}_M^T(:,k)\right]^T \in \mathcal{C}^{T\times 1}$, $\{\mathbf{N}_m\}_{m=1}^{M} = \mathbf{W}_m\mathbf{U}\mathbf{n}_m\mathbf{s}_m^H$, $\mathbf{W} = \left[\mathbf{W}_1^T, \mathbf{W}_2^T, \cdots, \mathbf{W}_M^T\right]^T$.

Note that in (7), there are three channels, $\mathbf{H}_{\mathrm{t}}$, $\mathbf{h}_{\mathrm{r},k}$, and $\mathbf{h}_{\mathrm{d},k}$ need to be estimated. And we also notice that the channel $\mathbf{h}_{\mathrm{d},k}$ is independent with the other two channels. Thus, we propose a two-stage channel estimation scheme, where we first estimate the channel $\mathbf{h}_{\mathrm{d},k}$, and then estimate the channel $\mathbf{H}_{\mathrm{t}}$ and $\mathbf{h}_{\mathrm{r},k}$ by removing the influence of $\mathbf{h}_{\mathrm{d},k}$. With setting the IRS to the absorbing mode, where $\boldsymbol{\Theta} = \mathbf{0}_{N\times N}$, the measurement vector $\tilde{\mathbf{Z}}_k^d \in \mathcal{C}^{T\times 1}$ for the channel $\mathbf{h}_{\mathrm{d},k}$ can be obtained as

$$\tilde{\mathbf{Z}}_k^d = \mathbf{W}\mathbf{U}\mathbf{h}_{\mathrm{d},k} + \mathbf{N}(:,k) \tag{8}$$

Since (8) is a traditional channel estimation scheme, we can estimate the channel $\mathbf{h}_{\mathrm{d},k}$ with traditional solutions, such as [12]. After that, removing the influence $\tilde{\mathbf{Z}}_k^d$ from $\tilde{\mathbf{Z}}_k$, the residual measurement vector $\tilde{\mathbf{Z}}_k^r \in \mathcal{C}^{T\times 1}$ for the channel $\mathbf{H}_{\mathrm{t}}$ and $\mathbf{h}_{\mathrm{r},k}$ can be given as

$$\tilde{\mathbf{Z}}_k^r = \tilde{\mathbf{Z}}_k - \tilde{\mathbf{Z}}_k^d = \mathbf{W}\mathbf{U}\mathbf{H}_{\mathrm{t}}\boldsymbol{\Theta}\mathbf{h}_{\mathrm{r},k} + \mathbf{N}(:,k) \tag{9}$$

To estimate the channel $\mathbf{H}_{\mathrm{t}}$ and $\mathbf{h}_{\mathrm{r},k}$ in (9), we set the IRS to the perfect reflecting mode, where $\boldsymbol{\Theta} = \mathbf{I}_{N\times N}$. Then, (9) can be rewritten as

$$\tilde{\mathbf{Z}}_k^r = \mathbf{W}\mathbf{U}\mathbf{H}_{\mathrm{t}}\mathbf{h}_{\mathrm{r},k} + \mathbf{N}(:,k) \tag{10}$$

Although it is able to estimate the cascaded channel $\mathbf{H}_{\mathrm{t}}\mathbf{h}_{\mathrm{r},k}$ in (10), it is hard to separate $\mathbf{H}_{\mathrm{t}}\mathbf{h}_{\mathrm{r},k}$ into $\mathbf{H}_{\mathrm{t}}$ and $\mathbf{h}_{\mathrm{r},k}$. Fortunately, for the MISO system, we do not need to separately estimate $\mathbf{H}_{\mathrm{t}}$ and $\mathbf{h}_{\mathrm{r},k}$, since we have

$$\mathbf{H}_{\mathrm{t}}\boldsymbol{\Theta}\mathbf{h}_{\mathrm{r},k} = \mathbf{H}_{\mathrm{t}}\,\mathrm{diag}\left(\mathbf{h}_{\mathrm{r},k}\right)\mathbf{v} \tag{11}$$

where $\mathbf{v} = \left[e^{j\theta_1}, e^{j\theta_2}, \cdots, e^{j\theta_N}\right]^T \in \mathcal{C}^{N\times 1}$ is a $N\times 1$ vector consisting of the diagonal elements of $\boldsymbol{\Theta}$. Therefore, for any data rate optimization problems in the MISO system, the IRS can optimize the phase-shifts by only knowing the channel $\mathbf{H}_{\mathrm{t}}\,\mathrm{diag}\left(\mathbf{h}_{\mathrm{r},k}\right)$. Thus, we have

$$\begin{aligned}\tilde{\mathbf{Z}}_k^r &= \mathbf{W}\mathbf{U}\mathbf{H}_{\mathrm{t}}\mathbf{h}_{\mathrm{r},k} + \mathbf{N}(:,k)\\ &= \mathbf{W}\mathbf{U}\mathbf{H}_{\mathrm{t}}\,\mathrm{diag}\left(\mathbf{h}_{\mathrm{r},k}\right)\mathbf{1}_{N\times 1} + \mathbf{N}(:,k)\end{aligned} \tag{12}$$

To estimate the channel $\mathbf{H}_{\mathrm{t}}\,\mathrm{diag}\left(\mathbf{h}_{\mathrm{r},k}\right)$ in (12), we denote the estimated $\tilde{\mathbf{Z}}_k^d$ as $\hat{\mathbf{Z}}_k^d$, and the estimated $\tilde{\mathbf{Z}}_k^r$ as $\hat{\mathbf{Z}}_k^r$. Then problem (5) can be reformulated as

$$\min_{\hat{\mathbf{H}}_{\mathrm{d}},\hat{\mathbf{H}}_{\mathrm{t}}\,\mathrm{diag}\left(\mathbf{h}_{\mathrm{r},k}\right)} \sum_{k=1}^{K}\left\|\tilde{\mathbf{Z}}_k^d - \hat{\mathbf{Z}}_k^d\right\|_F^2 + \sum_{k=1}^{K}\left\|\tilde{\mathbf{Z}}_k^r - \hat{\mathbf{Z}}_k^r\right\|_F^2 \tag{13}$$

To solve problem (13), we first propose a proposition to prove a special property of the IRS beamspace channel, which is the base of our proposed channel estimation scheme.

Proposition 1: Denote the effective channel $\mathbf{h}_k^{\mathrm{eff}} = \mathbf{U}\mathbf{H}_{\mathrm{t}}\mathbf{h}_{\mathrm{r},k}$ as $\mathbf{h}_k^{\mathrm{eff}} = \sqrt{\frac{N_t N^2}{L_t L_r}}\sum_{i=1}^{L_t}\mathbf{c}_{k,i}$, where $\left\{\mathbf{c}_{k,i}\right\}_{i=1}^{L_t} \in \mathcal{C}^{N_t\times 1}$ is the ith channel component of $\mathbf{h}_k^{\mathrm{eff}}$. When the number of transmission antennas N_t tends to infinity, we have

$$\lim_{N\to\infty}\left|\mathbf{c}_{k,i}^H\mathbf{c}_{k,i}\right| = 0, \quad \forall i, j = 0,1,\cdots,L_t, i\neq j \tag{14}$$

which means that any two channel components $\mathbf{c}_{k,i}$ and $\mathbf{c}_{k,j}$ in $\mathbf{h}_k^{\mathrm{eff}}$ are independent.

Proof: Based on (3), the ith channel component $\mathbf{c}_{k,i}$ can be presented as

$$
\begin{aligned}
\mathbf{c}_{k,i} &= \mathbf{U}\alpha_i \mathbf{a}_t\left(\psi_i^t\right)\mathbf{a}_r^H\left(\psi_i^r\right)\sum_{j=1}^{L_r}\alpha_j \mathbf{a}_t\left(\psi_j^t\right) \\
&= \alpha_i \mathbf{U}\sum_{j=1}^{L_r}\alpha_j \mathbf{a}_t\left(\psi_i^t\right)\mathbf{a}_r^H\left(\psi_i^r\right)\mathbf{a}_t\left(\psi_j^t\right) \\
&= \left[\alpha_i\sum_{j=1}^{L_r}\alpha_j \mathbf{a}^H\left(\bar{\psi}_1\right)\mathbf{a}_t\left(\psi_i^t\right)\mathbf{a}_r^H\left(\psi_i^r\right)\mathbf{a}_t\left(\psi_j^t\right),\cdots,\alpha_i\sum_{j=1}^{L_r}\alpha_j \mathbf{a}^H\left(\bar{\psi}_{N_t}\right)\mathbf{a}_t\left(\psi_i^t\right)\mathbf{a}_r^H\left(\psi_i^r\right)\mathbf{a}_t\left(\psi_j^t\right)\right]^T \\
&= \left[\alpha_i\sum_{j=1}^{L_r}\alpha_j \frac{\sin\left[N_t\pi\left(\psi_i^t-\bar{\psi}_1\right)\right]}{N_t\sin\left[\pi\left(\psi_i^t-\bar{\psi}_1\right)\right]}\frac{\sin\left[N\pi\left(\psi_j^t-\psi_i^r\right)\right]}{N\sin\left[\pi\left(\psi_j^t-\psi_i^r\right)\right]},\cdots\right. \\
&\qquad\qquad \left.,\alpha_i\sum_{j=1}^{L_r}\alpha_j \frac{\sin\left[N_t\pi\left(\psi_i^t-\bar{\psi}_{N_t}\right)\right]}{N_t\sin\left[\pi\left(\psi_i^t-\bar{\psi}_{N_t}\right)\right]}\frac{\sin\left[N\pi\left(\psi_j^t-\psi_i^r\right)\right]}{N\sin\left[\pi\left(\psi_j^t-\psi_i^r\right)\right]}\right]^T \\
&= \left[\alpha_i\sum_{j=1}^{L_r}\alpha_j\Xi\left(N_t,\psi_i^t-\bar{\psi}_1\right)\Xi\left(N,\psi_j^t-\psi_i^r\right),\cdots,\alpha_i\sum_{j=1}^{L_r}\alpha_j\Xi\left(N_t,\psi_i^t-\bar{\psi}_{N_t}\right)\Xi\left(N,\psi_j^t-\psi_i^r\right)\right]^T \\
&= \left[\alpha_i\Xi\left(N_t,\psi_i^t-\bar{\psi}_1\right)\sum_{j=1}^{L_r}\alpha_j\Xi\left(N,\psi_j^t-\psi_i^r\right),\cdots,\alpha_i\Xi\left(N_t,\psi_i^t-\bar{\psi}_{N_t}\right)\sum_{j=1}^{L_r}\alpha_j\Xi\left(N,\psi_j^t-\psi_i^r\right)\right]^T \\
&= \left[\bar{\alpha}_i\Xi\left(N_t,\psi_i^t-\bar{\psi}_1\right),\cdots,\bar{\alpha}_i\Xi\left(N_t,\psi_i^t-\bar{\psi}_{N_t}\right)\right]^T,
\end{aligned}
$$

where $\bar{\alpha}_i=\alpha_i\sum_{j=1}^{L_r}\alpha_j\Xi\left(N,\psi_j^t-\psi_i^r\right)$, $\Xi\left(\bar{N},\psi\right)=\frac{\sin\left[\bar{N}\pi\psi\right]}{\bar{N}\sin\left[\pi\psi\right]}$, and $x=1,2,\cdots,N_t$. And it has been demonstrated in [13] when $\mathbf{c}_{k,i}$ is formulated as (a), **Proposition 1** can be true.

According to **Proposition 1**, we can estimate the channel $\mathbf{U}\mathbf{H}_{\mathrm{t}}\mathbf{h}_{\mathrm{r},k}$ by estimating a series of independent channel components $\left\{\hat{\mathbf{c}}_{k,i}\right\}_{i=1}^{L_t}$, which can decrease the complexity of the channel estimation. When $\hat{\mathbf{H}}_{\mathrm{t}}\hat{\mathbf{h}}_{\mathrm{r},k}$ is obtained, $\hat{\mathbf{H}}_{\mathrm{t}}\,\mathrm{diag}\left(\hat{\mathbf{h}}_{\mathrm{r},k}\right)$ is able to be calculated by $\hat{\mathbf{H}}_{\mathrm{t}}\,\mathrm{diag}\left(\hat{\mathbf{h}}_{\mathrm{r},k}\right)=\hat{\mathbf{H}}_{\mathrm{t}}\hat{\mathbf{h}}_{\mathrm{r},k}\left(\mathbf{1}_{N\times 1}\right)^{+}$, where $(\cdot)^{+}$ is the pseudo inverse operation.

Summarizing the above analysis, the detailed steps of our proposed two-stage channel estimation scheme are illustrated in Algorithm 1. Specifically, in stage 1, we estimate $\mathbf{h}_{\mathrm{d},k}$ by setting the IRS to the absorbing mode. And in stage 2, we estimate $\mathbf{H}_{\mathrm{t}}\,\mathrm{diag}\left(\mathbf{h}_{\mathrm{r},k}\right)$ by setting the IRS mode to the perfect reflecting mode. In step 3, we remove the influence of the estimated channel $\hat{\mathbf{h}}_{\mathrm{d},k}$, and obtain the measurement vector $\tilde{\mathbf{Z}}_k^r$ for $\mathbf{H}_{\mathrm{t}}\mathbf{h}_{\mathrm{r},k}$. Then, we enumerate $\mathbf{U}\mathbf{H}_{\mathrm{t}}\mathbf{h}_{\mathrm{r},k}$ by estimating its channel component $\left\{\mathbf{c}_{k,i}\right\}_{i=1}^{L_t}$ one by one. For the i th component, in step 5, we detect the position of the strongest element of $\mathbf{c}_{k,i}$. And in step 6, we construct a position vector $\gamma_i = \left[n_i^* - V, \cdots, n_i^* + V\right]$, which represents the position of the most $2V+1$ strong elements in $\mathbf{c}_{k,i}$, since for the function $\Xi\left(N_t, \psi_i^t - \bar{\psi}_n\right)$ in **Proposition 1**, when n is more close to n_i^*, $\Xi\left(N_t, \psi_i^t - \bar{\psi}_n\right)$ can be more close to $\Xi\left(N_t, \psi_i^t - \bar{\psi}_{n_i^*}\right)$. Next, we use the $2V+1$ elements to extract the sub measurement vector $\tilde{\mathbf{Z}}_{k,i}^r \in \mathcal{C}^{2V+1}$ from $\tilde{\mathbf{Z}}_k^r$ as $\tilde{\mathbf{Z}}_{k,i}^r = \tilde{\mathbf{Z}}_{k,i}^r(\gamma_i, :)$, where $\tilde{\mathbf{Z}}_k^r$ can be approximated by $\tilde{\mathbf{Z}}_{k,i}^r$ since we have $\left\|\tilde{\mathbf{Z}}_k^r\right\|_F^2 \approx \left\|\tilde{\mathbf{Z}}_{k,i}^r\right\|_F^2$. With the reduced-dimensional measurement vector $\tilde{\mathbf{Z}}_{k,i}^r$, in step 7, we use a least square method to estimate $\mathbf{c}_{k,i}(\gamma_i, :)$ by $\hat{\mathbf{c}}_{k,i}(\gamma_i, :) = \left[\mathbf{W}(:, \gamma_i)\right]^+ \tilde{\mathbf{Z}}_{k,i}^r$, which can effectively reduce the computational complexity. After all L_t channel components are estimated, in step 10, we can obtain $\mathbf{U}\hat{\mathbf{H}}_\mathrm{t}\hat{\mathbf{h}}_{\mathrm{r},k}$ as $\mathbf{U}\hat{\mathbf{H}}_\mathrm{t}\hat{\mathbf{h}}_{\mathrm{r},k} = \sum_{i=1}^{L_t} \hat{\mathbf{c}}_{k,i}$. And $\hat{\mathbf{H}}_\mathrm{t}\,\mathrm{diag}\left(\hat{\mathbf{h}}_{\mathrm{r},k}\right)$ can be estimated in the end.

Algorithm 1: Proposed Two-Stage Channel Estimation Scheme

Require: $\tilde{\mathbf{Z}}_k^r, \tilde{\mathbf{Z}}_k^d, \mathbf{W}, V, L_t$

1: Stage 1: Set the absorbing mode and estimate $\mathbf{h}_{\mathrm{d},k}$

2: Stage 2: Set the IRS to the perfect reflecting mode

3: Compute $\tilde{\mathbf{Z}}_k^r = \tilde{\mathbf{Z}}_k - \tilde{\mathbf{Z}}_k^d$, and set $\left\{\mathbf{c}_{k,i}\right\}_{i=1}^{L_t} = \mathbf{0}_{N_t \times 1}$

4: **for** $1 \le i \le L_t$ **do**

5: Detect $n_i^* = \underset{1\le n \le N_t}{\arg\max} \left|\mathbf{W}^H(:, n)\tilde{\mathbf{Z}}_k^r\right|$

6: Select $\tilde{\mathbf{Z}}_{k,i}^r = \tilde{\mathbf{Z}}_{k,i}^r(\gamma_i, :)$, $\gamma_i = \left[n_i^* - V, \cdots, n_i^* + V\right]$

7: Estimate $\hat{\mathbf{c}}_{k,i}(\gamma_i, :) = \left[\mathbf{W}(:, \gamma_i)\right]^+ \tilde{\mathbf{Z}}_{k,i}^r$

8: Calculate $\tilde{\mathbf{Z}}_k^r = \tilde{\mathbf{Z}}_k^r - \tilde{\mathbf{Z}}_{k,i}^r$

9: **end for**

10: $\mathbf{U}\hat{\mathbf{H}}_\mathrm{t}\hat{\mathbf{h}}_{\mathrm{r},k} = \sum_{i=1}^{L_t} \hat{\mathbf{c}}_{k,i}$

Ensure: $\hat{\mathbf{h}}_{\mathrm{d},k}$, $\hat{\mathbf{H}}_\mathrm{t}\,\mathrm{diag}\left(\hat{\mathbf{h}}_{\mathrm{r},k}\right) = \hat{\mathbf{H}}_\mathrm{t}\hat{\mathbf{h}}_{\mathrm{r},k}\left(\mathbf{1}_{N\times 1}\right)^+$

4. Numerical results

In this section, numerical results are presented to demonstrate the performance of Algorithm 1. We consider an IRS aided THz multi-user MISO system, where the BS equips with a lens antenna array with $N_t = 256$ antennas, simultaneously serves to $K = 16$ single-antenna users with the aid of an IRS, which is with $N = 16$ reflecting elements. For the channel $\mathbf{H}_\mathrm{t}$ and

$\{\mathbf{h}_{\mathrm{r},k}\}_{k=1}^{K}$, we assume the complex gain is $\alpha \in \mathcal{CN}(0,1)$, the spatial direction ψ follows a uniform distribution within $[-0.5, 0.5]$, and $L_t = L_r = 3$ due to the sparsity of the THz channel. For simplicity, we assume $\{\mathbf{h}_{\mathrm{r},k}\}_{k=1}^{K} = \mathbf{0}_{N_t \times 1}$ which means that the directly transmissions between the BS and the user are broken down with some obstacles. Finally, all the results are averaged over 5000 random channel realizations.

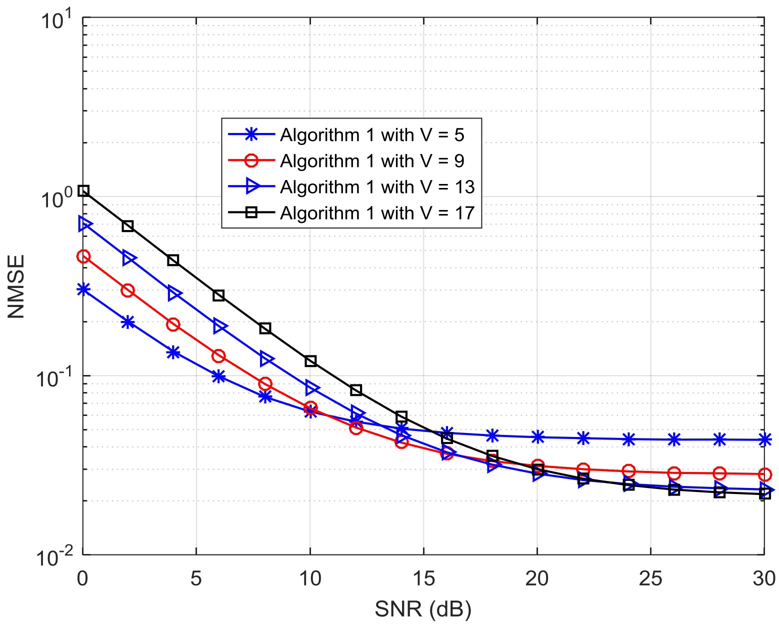

Fig. 2 Normalized mean square error comparison versus SNR.

Fig. 2 shows the normalized mean square error (NMSE) performance versus SNR in different schemes, where we define

$$\mathrm{NMSE} = \frac{\sum_{k=1}^{K} \left\| \mathbf{U}\mathbf{H}_{\mathrm{t}} \operatorname{diag}\left(\mathbf{h}_{\mathrm{r},k}\right) - \mathbf{U}\hat{\mathbf{H}}_{\mathrm{t}} \operatorname{diag}\left(\hat{\mathbf{h}}_{\mathrm{r},k}\right) \right\|_F^2}{\sum_{k=1}^{K} \left\| \mathbf{U}\mathbf{H}_{\mathrm{t}} \operatorname{diag}\left(\mathbf{h}_{\mathrm{r},k}\right) \right\|_F^2}.$$

As shown in Fig. 2, the NMSE performance can be improved with the increasing SNR. Therefore, when the SNR is smaller than 10 *dB*, the NMSE can be lower when V decreases, but when the SNR is larger than 10 *dB*, the result will be reversed.

5. Conclusions

In this letter, we investigate the channel estimation problem for the IRS-aided multi-user MISO system with lens antenna array. Specifically, we propose a two-stage channel estimation scheme, where we first have estimated the channel without IRS by setting the IRS to the absorbing mode, and then we have estimated the cascaded channel reflected by the IRS with removing the influence of the prior estimated channel. Since we demonstrate that the channel components of the cascaded channel are independent, in stage 2, we decompose the total channel estimation problem into a series of independent problems, where we have estimated each channel component by the least square method. Numerical results show the effectiveness of our proposed channel estimation scheme.

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