Invited Paper

Continuously frequency-tunable 0.22 THz gyrotron oscillator with quasi-optical resonator

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(Received 13 December 2016)

Abstract: A 0.22 *THz* continuously frequency-tunable gyrotron oscillator with quasi-optical resonator has been investigated in this paper. The operating frequency is changed by mechanically adjusting the mirror separation. The simulation results have demonstrated that an 11.7 *GHz* bandwidth of frequency tuning and output power of 38 *kW* could be achieved.

Keywords: Cylindrical confocal waveguide, Frequency tunable, Quasi-optical resonator, THz gyrotron

doi: 10.11906/TST.166-176.2016.12.16

1. Introduction

THz gyrotron has attracted great attentions due to its ability of high output power and has been applied in fields of plasma heating, nondestructive testing, and high resolution radar [2]1-3]. Conventional gyrotron oscillators usually employ resonator of high quality factor and operate at single frequency or discrete frequencies for different modes. However, for some emerging applications such as the Nuclear Magnetic Resonance (NMR) spectroscopy enhanced by Dynamic Nuclear Polarization (DNP) [4], radiation sources are required to be continuously tunable in a wide frequency range.

Several approaches of frequency tuning for gyrotron oscillator have been suggested, one based on exciting series of high order longitudinal modes and 2 *GHz* tuning range are obtained [6], but the output power is just in the order of several watts. There is another approach to make gyrotron oscillator continuously tunable by splitting the cavity into two semicircle parts and varying the slit width [7].Frequency tuning could also be realized with coaxial gyrotron by changing the ratio of the radiuses of external and internal conductors mechanically [8]. In this paper, we have investigated a broadband continuously frequency-tunable gyrotron with quasi-optical resonator based on open confocal waveguide.This resonator has high power capacity due to its large size and good mode selection, which has been demonstrated to be a qualified beam-wave interaction structure for gyrotron oscillator [9] and gyro-traveling-wave amplifier [10]. The Particle-In-Cell (PIC) simulation results demonstrate that, by adjusting the distance between two mirrors, wide and smooth frequency tuning with high output power can be achieved.

2. Characteristics of quasi-optical resonator

The resonator consists of three parts, as shown in Fig. 1, a straight section, a linearly down-tapered section at the entrance and a linearly up-tapered section at the output.

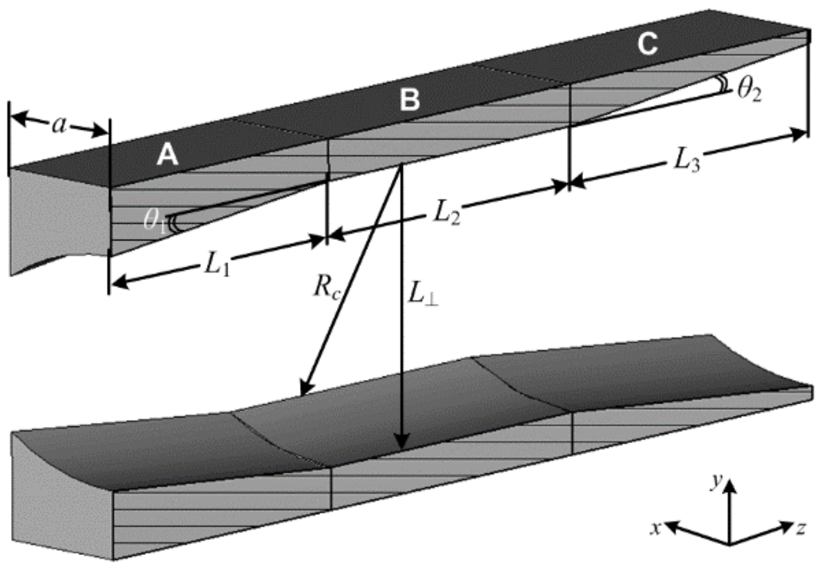

Fig. 1 Schematic diagram of quasi-optical resonator.

The membrane function $\psi(\xi, \zeta)$ for TE_{mn} mode in a confocal waveguide can be expressed as [9-11]

$$\psi_{mn}(\xi,\zeta) = C_m \frac{H_m\left(\sqrt{2\gamma}\xi\right)}{\sqrt{\cosh\zeta}} e^{-\frac{\gamma\xi^2}{2}} \cdot \begin{cases} \cos\phi(\zeta), & n = 1,3,5\cdots \\ j\sin\phi(\zeta), & n = 2,4,6\cdots \end{cases} \tag{1}$$

$$\phi(\zeta) = \gamma\sinh\zeta - \left(m + \frac{1}{2}\right)\arcsin(\tanh\zeta)$$

where C_m is a complex constant, m and n are the mode indexes referring to the number of field variations in x and y direction respectively, and $H_m(\tau)$ is the mth Hermite polynomial defined as

$$\begin{cases} H_m(\tau) = (-1)^m e^{\frac{\tau^2}{2}} \dfrac{d^m}{d\tau^m}\left(e^{-\frac{\tau^2}{2}}\right) \\ H_0(\tau) = 1, H_1(\tau) = \tau, H_2(\tau) = \tau^2 - 1, \cdots \end{cases} \tag{2}$$

The fields are

$$\begin{cases} H_z(x,y) = k_{mn}^2 \psi_{mn}(x,y) \\ H_x(x,y) = jk_z f(z) \dfrac{\partial \psi(x,y)}{\partial x} \\ H_y(x,y) = jk_z f(z) \dfrac{\partial \psi(x,y)}{\partial y} \end{cases} \qquad \begin{cases} E_x(x,y) = j\omega\mu f(z) \dfrac{\partial \psi(x,y)}{\partial y} \\ E_y(x,y) = -j\omega\mu f(z) \dfrac{\partial \psi(x,y)}{\partial x} \\ E_z(x,y) = 0 \end{cases} \tag{3}$$

where k_{mn} and k_z are the transverse and longitudinal propagation constants, respectively, and $k_{mn}^2 + k_z^2 = k^2$.

The k_{mn} and the cut-off frequency f_{cmn} can be obtained:

$$k_{mn} = \frac{\pi}{L_\perp}\left(n + \frac{2m+1}{\pi}\arcsin\sqrt{\frac{L_\perp}{2R_c}}\right) \tag{4}$$

$$f_{cmn} = \frac{k_{mn}c}{2\pi} = \frac{c}{2L_\perp}\left(n + \frac{2m+1}{\pi}\arcsin\sqrt{\frac{L_\perp}{2R_c}}\right) \tag{5}$$

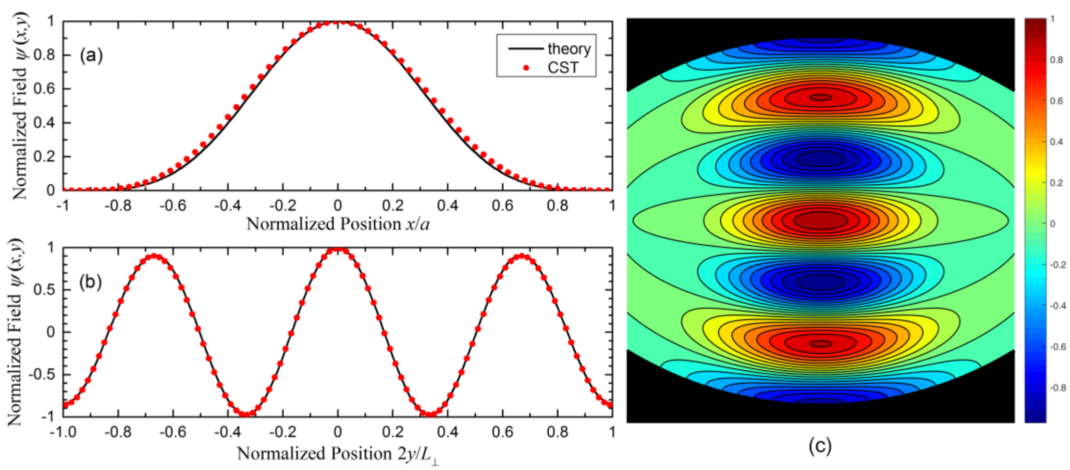

Fig. 2 Membrane function $\psi(x, y)$ for TE_{06} mode: (a) x-direction distribution (y = 0); (b) y-direction distribution (x = 0). The solid lines are the theoretical results calculated by eq.(1) and the pointed markers are the simulation results of CST Microwave Studio. ($R_c = L_\perp$=4.2 *mm*, 2a=4.4 *mm*)

Numerical results of the membrane function $\psi(x, y)$ for TE_{06} mode are given in Fig. 2. It can be seen that the fields in quasi-optical waveguide are Hermite-Gaussian distribution both in x and y directions. Fig. 2 also shows a good agreement between theoretical results and simulation results of CST Microwave Studio.

According to eq.(5), it is obvious that the cut-off frequency of quasi-optical waveguide depends on the mirror separation $L_\perp$, therefore, it is possible to continuously change the cut-off frequency of the operating mode.Consequently, the frequency of gyrotron oscillator would be changed by mechanically adjusting the mirror separation $L_\perp$.

Considering the diffraction losses, the transverse wavenumber $k_\perp$ is complex, $k_\perp = k_{mn} + j\Lambda/(2L_\perp)$, where the parameter Λ can be obtained by simplified numerical interpolation 错误!未找到引用源。12, 13]

$$\log_{10}\Lambda = \begin{cases} -0.0069C_F^2 - 0.7088C_F + 0.5443, & m = 0 \\ -0.0226C_F^2 - 0.4439C_F + 1.0820, & m = 1 \\ -0.0363C_F^2 - 0.1517C_F + 1.0075, & m = 2 \end{cases} \tag{6}$$

where $C_F = k_{\perp r}a^2/L_\perp$ is the Fresnel parameter, then the diffraction loss rate of confocal waveguide can be written as

$$LossRate(dB/cm) = 20\log_{10}\left[\exp\left(\frac{k_{zi}}{100}\right)\right] \tag{7}$$

where $k_{zi} = \mathrm{Im}\left[\sqrt{(\omega/c)^2 - k_\perp^2}\right]$.

In confocal waveguide, the diffraction loss of modes with $m = 0$ is much smaller than modes with $m \neq 0$, so the TE_{0n} mode should be chosen as operating mode , and TE_{06} is selected in this paper. Then the mirror radius R_c is set to be 4.2*mm*, the corresponding cut-off frequency is 223.06*GHz*, and the mirror aperture is 4.4*mm* to ensure TE_{06} mode which has a much smaller diffraction loss than parasitic modes. The parameters for the quasi-optical cavity are listed in Table 1.

Tab. 1 Parameters for the Quasi-Optical Resonator

L_1	10 *mm*
θ_1	2.1°
L_2	13 *mm*
Mirror Radius (R_c)	4.20 *mm*
Mirror Separation ($L_\perp$)	4.10~4.40 *mm*
L_3	10 *mm*
θ_2	0.86°

Assuming the longitudinal component of magnetic field in cold cavity with following forms:

$$H_z\left(x,y,z\right)=\psi\left(x,y\right)f\left(z\right) \tag{8}$$

where $f(z)$ is the field profile and satisfies the following equation [14].

$$\frac{d^2 f\left(z\right)}{d^2 z}+k_z^2\left(z\right)f\left(z\right)=0 \tag{9}$$

and the boundary conditions at left end ($z=z_{in}$) and right end ($z=z_{out}$) become

$$\left.\frac{df\left(z\right)}{dz}\right|_{z=z_{in}}-jk_z\left(z_{in}\right)f\left(z_{in}\right)=0 \tag{10}$$

$$\left.\frac{df\left(z\right)}{dz}\right|_{z=z_{out}}+jk_z\left(z_{out}\right)f\left(z_{out}\right)=0 \tag{11}$$

Using numerical method to solve (9)-(11), $f(z)$ and the complex oscillation frequency f_{osc} could be obtained. Hence, the axial diffraction quality value of the resonator $Q_{diff\parallel}$ is given by

$$Q_{diff\parallel}=\frac{\mathrm{Re}\left(f_{osc}\right)}{2\,\mathrm{Im}\left(f_{osc}\right)} \tag{12}$$

while the transverse diffraction quality factor $Q_{diff\perp}$ is defined by the diffraction loss parameter Λ [9]

$$Q_{diff\perp}=\frac{k_{\perp r}L_{\perp}}{\Lambda} \tag{13}$$

For metallic waveguide walls, the ohmic losses are characterized by ohmic quality factor Q_{ohm}

$$Q_{ohm}=\frac{2}{\delta}\frac{\iiint_V\left|H\right|^2 dV}{\iint_{S_{mirror}}\left|H_{\perp}\right|^2 dS} \tag{14}$$

where δ is the skin depth of metallic walls. Therefore, the total quality factor of the resonator Q_{total} satisfies

$$\frac{1}{Q_{total}}=\frac{1}{Q_{diff\parallel}}+\frac{1}{Q_{diff\perp}}+\frac{1}{Q_{ohm}} \tag{15}$$

Q_{ohm} for TE_{06} mode at 223*GHz* is calculated to be about 29500, assuming copper mirrors of resistivity ρ = $1.72\times10^{-8}\Omega\cdot m$. Therefore, Q_{total} mainly depends on the diffraction quality factor

Q_{diff}.

The dependences of oscillation frequency f_{osc} and diffraction quality factor Q_{diff} of TE_{06} mode on the mirror separation $L_{\perp}$ are shown in Fig. 3 with the mirror radius R_c = 4.2*mm*. It is seen that the oscillation frequency decreases from 228.6*GHz* to 213.4*GHz* when the mirror separation increases from 4.10*mm* to 4.40*mm*. At the same time, the diffraction quality factor decreases from 3470 to 2510.

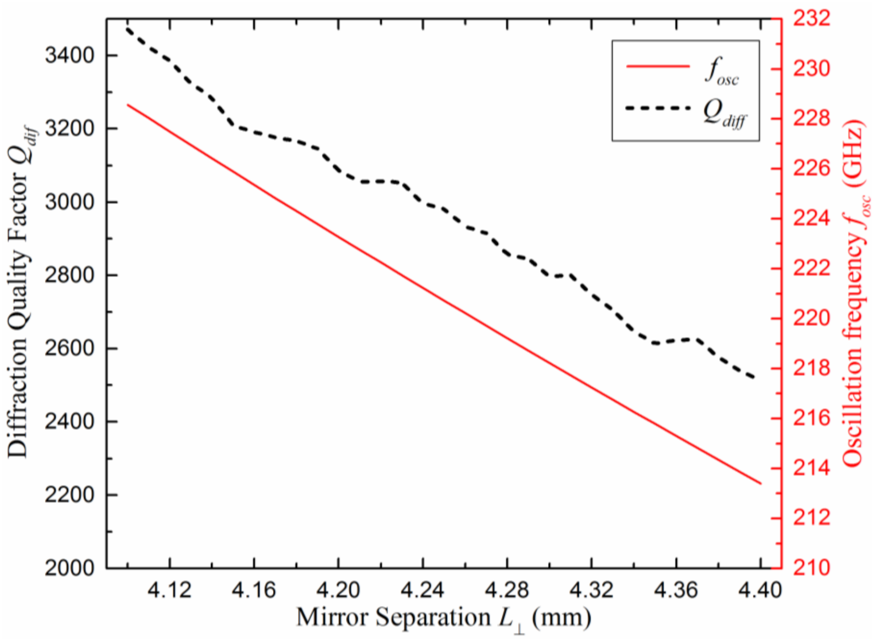

Fig. 3 Dependences of f_{osc} and Q_{diff} on the mirror separation $L_{\perp}$

3. Beam-wave interaction in quasi-optical cavity

According to theory of gyrotron oscillator, the beam-wave interaction is characterized by the interaction coupling coefficient [15]

$$G = \frac{\left\langle \left| M_1 \right|^2 \right\rangle \lambda^2}{16\pi \int_{S_\perp} \left| \psi(x,y) \right|^2 dxdy} \tag{16}$$

where $S_\perp$ is area of cavity cross-section , and for an annular electron beam, the term of $\left\langle \left| M_1 \right|^2 \right\rangle$ is given by

$$\left\langle \left| M_1 \right|^2 \right\rangle = \frac{1}{2\pi} \int_0^{2\pi} \left| \frac{1}{k} \left(\frac{\partial}{\partial X} + i \frac{\partial}{\partial Y} \right) \psi(X,Y) \right|^2 d\varphi \tag{17}$$

where $X = r_b \cos\varphi, Y = r_b \sin\varphi$ and r_b is the radius of electron beam.

The dependence of coupling coefficient G on the electron beam radius r_b is shown in Fig. 4, which indicates that r_b = 1.15*mm* is the best choice to get high coupling efficiency for TE_{06} mode.

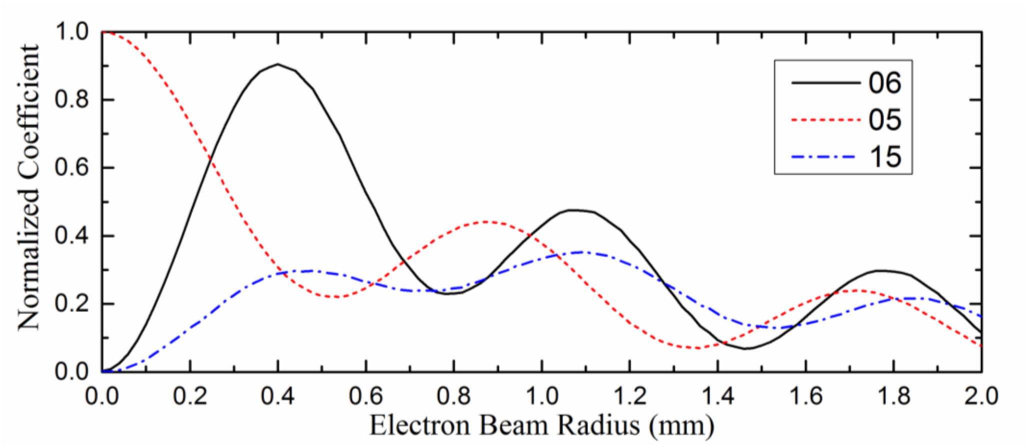

Fig. 4 Radial distribution of normalized coupling coefficient of TE_{05}, TE_{15} and TE_{06} modes.

The starting current I_{st} of a gyrotron with arbitrary cross section and an annular electron beam is expressed by [16]

$$I_{st} = 2^{-1/2}\pi^{3/2}\frac{\varepsilon_0 m_e c^3}{e}\frac{e^{2x^2}}{\mu^2(\mu x - s)}\left(\frac{L}{\lambda}\right)\left(\frac{2^s s!}{s^s}\right)^2\frac{\gamma_0\beta_{\perp 0}^{2(3-s)}}{GQ} \tag{18}$$

where

$$x = \frac{\mu\Delta}{4}, \mu = \frac{\pi L\beta_{\perp 0}^2}{\lambda\beta_{z0}}, \Delta = \frac{2}{\beta_{\perp 0}^2}\left(1 - \frac{s\Omega_0}{\omega\gamma_0}\right) \tag{19}$$

$$\beta_{\perp 0} = v_{\perp 0}/c, \beta_{z0} = v_{z0}/c \tag{20}$$

The starting current I_{st} for different modes are evaluated as function of magnetic field B_0 with fixed electron beam parameters (V_b=40*kV*, α=1.1, r_b=1.15*mm*) but different mirror separation $L_\perp$, the results are shown in Fig. 5. The minimum starting current for TE_{06} is about 0.75*A*, which is much lower than that of parasitic modes TE_{05} (2.2*A*) and TE_{15} (3.4*A*). In addition, as the mirror separation $L_\perp$ decreases, the minimum starting current becomes lower because of the increase of *Q*-value.

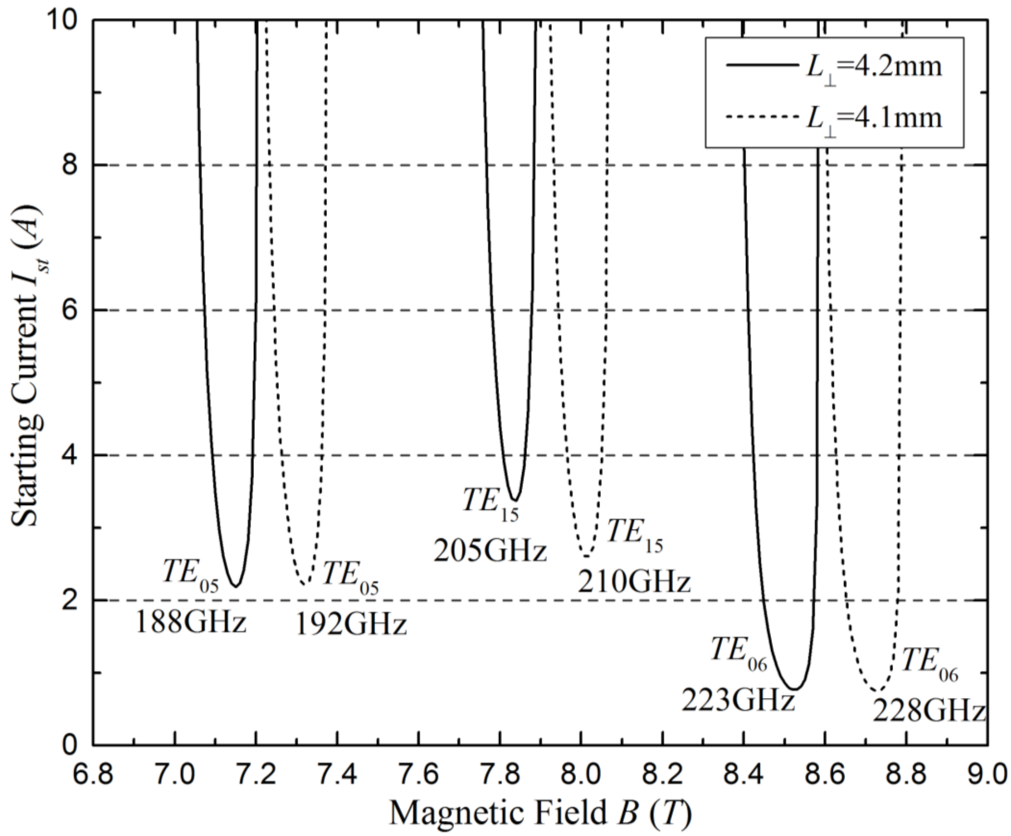

Fig. 5 Starting current I_{st} as function of magnetic field B_0

4. Results of PIC simulation

Tab. 2 Operating Parameters of the Quasi-Optical Gyrotron.

Beam Voltages (V_b)	40 *kV*
Beam Current (I_b)	5 A
Beam Pitch Factor (α)	1.1
Beam Radius (r_b)	1.15 *mm*
Magnetic Field (B_0)	8.4 *T*

According to the parameters listed in Table 1 and Table 2, a 0.22*THz* gyrotron oscillator with quasi-optical resonator has been simulated by 3D Particle-In-Cell code CHIPIC [17].When the straight section is a strictly confocal waveguide, $L_\perp = R_c = 4.2mm$, the simulation results show the gyrotron operating at TE_{06} mode and produces about 26*kW* output at 222.7*GHz*, as shown in Fig.6.

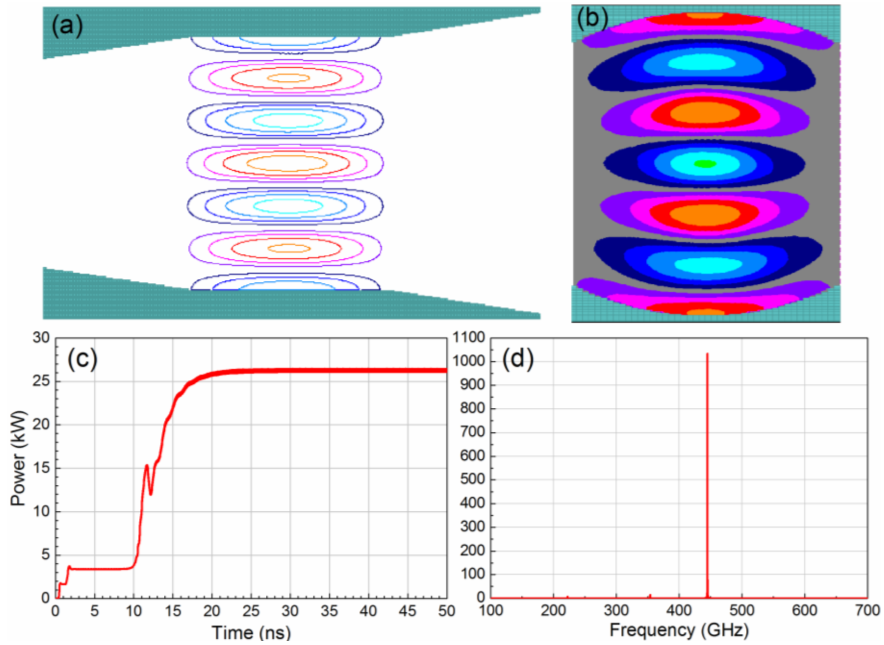

Fig. 6 Simulation results of the strictly confocal resonator, $L_{\perp} = R_c = 4.2$ *mm*. (a) Axial field contour of H_z at the middle plane; (b) Transverse field distribution of H_z at the output port; (c) Output power versus time; (d) Spectrum of output power.

To obtain the frequency tuning characteristics, $L_{\perp}$ is changed from 4.10*mm* to 4.40*mm* by moving the upper straight section along *y*-direction, while the other parts of the resonator are fixed, the oscillation frequency and output power, as function of mirror separation $L_{\perp}$, are illustrated in Fig. 7.

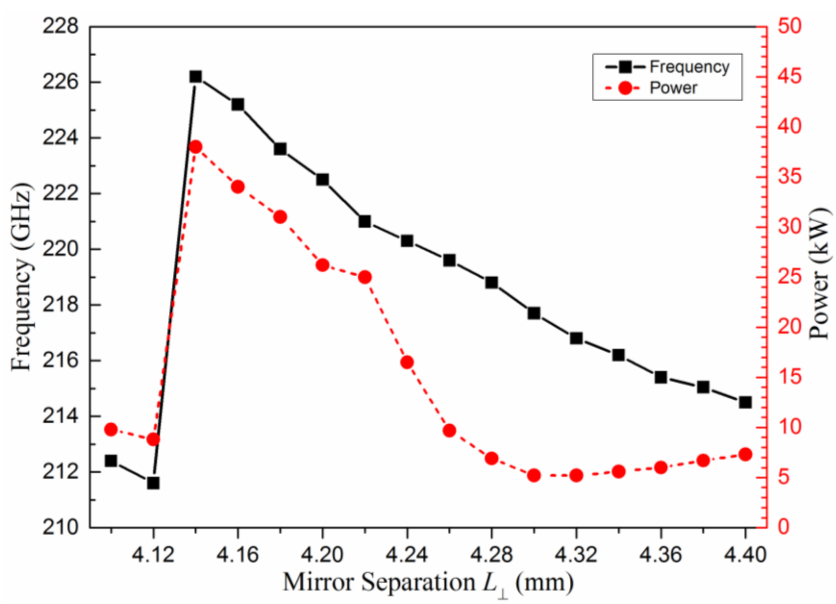

Fig. 7 Frequency and the output power versus the mirror separation $L_{\perp}$

We can see that the oscillation frequency continuously decreases from 226.2*GHz* to 214.5*GHz* when $L_\perp$ increases from 4.14*mm* to 4.40*mm*, and an 11.7*GHz* frequency tuning band (about 5.3% relative to the center frequency of 220*GHz*) is obtained. At the same time, the output power reduces from 38*kW* to 5*kW*. The PIC simulation results are in good agreement with the cold cavity results in Fig. 3. When $L_\perp$ is less than 4.14 *mm*, it is found that TE_{15} mode is easier to be excited than TE_{06} mode.

5. Conclusions

A continuously frequency tunable gyrotron oscillator with quasi-optical resonator has been investigated theoretically and been simulated with 3D PIC code. The simulation results indicate that the cylindrical confocal system has good mode-selective characteristics and could operate stably in high order mode. Moreover, the possibility of broadband continuously frequency tuning also has been demonstrated by PIC simulation, and an 11.7*GHz* bandwidth of oscillation frequency tuning from 214.5*GHz* to 226.2*GHz* is achieved by mechanically adjusting the mirror separation. According to the results of simulation, an experimental gyrotron has been designed and is being constructed now.

Acknowledgement

This work is supported by the Natural Science Foundation of China under Grant No. 61401064, the CAEP THz Science and Technology Foundation under the Grant No. CAEPTHZ201403, and the Specialized Research Fund for the Doctoral Program of Higher Education under Grant No. 20130185120027.

References

[1] Liu Shenggang. “Recent development of terahertz science and technology”. *China Basic Science*, 8(1), 7-12 (2006) (in Chinese).

[2] M. Thumm. “Recent advances in the worldwide fusion gyrotron development”. *IEEE Trans. Plasma Science*, 42, 3, 590–599 (2014).

[3] W. Fu, X. Guan, C. Chen, et al. “Design and Experiment of a 220/420-GHz Gyrotron for Nondestructive Evaluation”. *IEEE Trans. Electron Devices*, 61, 7, 2531–2537 (2014).

[4] E. A. Nanni, A. B. Barnes, R. G. Griffin, et al. “THz dynamic nuclear polarization NMR”. *IEEE Trans. Terahertz Sci. Technol*., 1, 1, 145–163 (2011).

[5] S. Sabchevski, T. Idehara, S. Mitsudo, et al. “Conceptual Design Study of a Novel Gyrotron for NMR/DNP Spectroscopy,” *Int J Infrared Milli Waves*, 26, 9, 1241-1264 (2005).

[6] C. Torrezan, M. A. Shapiro, J. R. Sirigiri, et al. "Operation of a continuously frequency-tunable second harmonic CW 330-*GHz* gyrotron for dynamic nuclear polarization". *IEEE Trans. Electron Devices*, 58, 8, 2777–2783 (2011).

[7] Bratman V. L., Kalynov Y. K., Kalynova G., et al., "Frequency Tuning in a Subterahertz Gyrotron with a Variable Cavity". *IEEE Trans. Electron Devices*, 61, 10, 3529-3533 (2014).

[8] M. Y. Glyavin, A. G. Luchinin, M. V. Morozkin, et al. "Smooth wideband tuning of the operating frequency of a gyrotron". *Radiophys. Quantum Electron*., 51, 1, 57–63 (2008).

[9] Hu W., Shapiro M., Kriescher K. E., et al. "140-*GHz* gyrotron experiments based on a confocal cavity". *IEEE Trans. Plasma Science*, 26, 3, 366-374 (1998).

[10] J. R. Sirigiri, M. A. Shapiro, and R. J. Temkin. "High-power 140-*GHz* quasioptical gyrotron traveling-wave amplifier". *Phys. Rev. Lett*., 90, 258302-1–258302-4 (2003).

[11] Weinstein, L. A., *Open Resonators and Open Waveguides*, Golem Press, Boulder, CO (1969).

[12] Joye C. D., Shapiro M., Sirigiri J. R., et al. "Demonstration of a 140-*GHz* 1-*kW* confocal gyro-traveling-wave amplifier". *IEEE Trans. Electron Devices*, 56, 5, 818-827 (2001).

[13] Boyd, G. D., and J. P. Gordon. "Confocal multimode resonator for millimeter through optical wavelength masers". *Bell System Technical Journal*, 40, 2, 489-508 (1961).

[14] Liu Shenggang, *Relativistic electronics*, Science Press, Beijing, (1987) (in Chinese).

[15] V. Gaponov, et al. "Powerful millimetre-wave gyrotrons". *International Journal of Electronics,* 51, 4, 277-302, (1981).

[16] G. Danly and R. J. Temkin. "Generalized nonlinear harmonic gyrotron theory". *Physics of Fluids*, 29, 2, 561-567 (1986).

[17] J. Zhou, D. Liu, C. Liao and Z. Li. "CHIPIC: an efficient code for electromagnetic PIC modeling and simulation". *IEEE Trans. Plasma Science*, 37, 10, 2002–2011 (2009).