Equivalent circuit method of π-mode frequency of rising-sun magnetron

Song Yue [1*, 2], Zhao-chuan Zhang [1], and Dong-ping Gao [1]
[1] Key Laboratory of High Power Microwave Sources and Technologies, Institute of Electronics, Chinese Academy of Sciences, Beijing 100190, China
[2] University of Chinese Academy of Sciences, 19 BeisihuanXilu, Beijing 100049, China
[*1] Email: yuessd@gmail.com

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Abstract: In this paper, equivalent circuit method is adopted to investigate π-mode frequency of rising-sun magnetron theoretically. The equivalent inductance and capacitance of resonant cavities are calculated with structure dimensions, and then π-mode frequency is obtained. Theoretical results are consistent well with that of simulations.

Keywords: Rising-sun magnetron, Π-mode, Equivalent inductance, Equivalent capacitance

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1. Introduction

Magnetron is a kind of efficient and economical high power microwave generators, which is widely used in both military and civilian field. For the stable operation of the magnetron, two techniques are adopted to increase the mode separation: straps are widely used for magnetrons whose wavelength is larger than 3 *cm*, while rising-sun structure is used for magnetrons whose wavelength is smaller than 3 *cm*. Rising-sun structures can avoid the machining problem of tiny straps and increase the quality factor of the cavity.

There are commonly three methods to calculate π-mode frequency of magnetron: field analysis, computer simulation and experimental measurement. Among them, the field equation method is very complex and it usually requires solving transcendental equations; computer simulation usually takes a long time, while the experimental test is not suitable to the pre-design and analysis of magnetrons. Equivalent circuit can estimate resonance frequency of magnetrons by their structure dimensions directly, and adjust their structure dimensions according to the relation between resonance frequency and structure dimensions [1-4].

2. Theoretical analysis

Cross section of an 18 cavities rising-sun magnetron is shown in Figure 1.

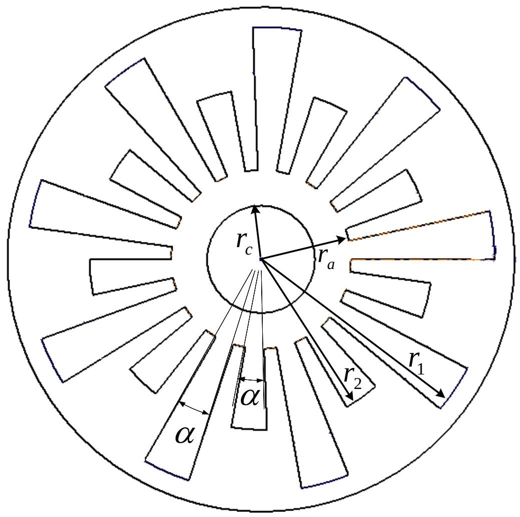

Fig. 1 Schematic of the cross section of rising-sun magnetron resonant system. r_c, r_a, r_1, *and* r_2 are radiuses of cathode, anode, large cavity and small cavity, respectively. α is the angular of sector cavity.

Resonant cavity can be represented by a parallel circuit of inductance and capacitance, so equivalent circuit of rising-sun magnetron resonant system is shown in Figure 2 [3].

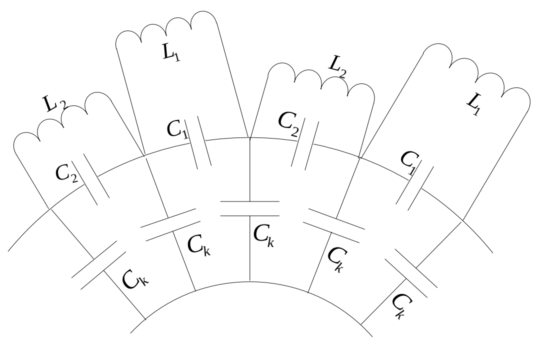

Fig. 2 Lumped-constant circuit of rising-sun magnetron. The circuit is supposed to continue around and close on itself and to contain an even number N of resonant circuits. L_1, C_1 are equivalent inductance and capacitance of large cavity, while L_2, C_2 are those of small cavity. C_k represents the coupling capacitances of the anode segments.

The expression of resonant frequency of rising-sun magnetron is [1]

$$\cos\varphi = 1 + \frac{C_k}{C_1}\frac{1}{1-\left(f_1/f\right)^2} + \frac{C_k}{C_2}\frac{1}{1-\left(f_2/f\right)^2} ? \frac{1}{2}\frac{C_k^{\,2}}{C_1C_2}\frac{1}{\left[1-\left(f_1/f\right)^2\right]\left[1-\left(f_2/f\right)^2\right]}, \qquad (1)$$

where f_1, f_2 are the resonant frequencies of large and small cavities, respectively. $\varphi=4\pi n/N$, (n=0,1,…,N/2) .

Coupling capacitance is given by [1]

$$C_k = \frac{\varepsilon_0 h \tau}{r_a \ln(r_a / r_c)}. \tag{2}$$

By expressions of equivalent inductance and capacitance of sector resonant cavity [4], equivalent inductance and capacitance of large cavity are

$$L_1 = \frac{\mu_0 \alpha D_1}{6h}\left(2r_1 + r_a\right), \tag{3}$$

and

$$C_1 = \frac{2\varepsilon_0 h}{D_1 \alpha}\left(r_1 \ln\frac{r_1}{r_a} - D_1\right) + \frac{\varepsilon_0 h \beta}{\left(\pi\text{-}\alpha\text{-}\beta\right)\tan\left(\beta/2 + \alpha/2\right)}, \tag{4}$$

where μ_0 is the permeability of vacuum, ε_0 is the permittivity of vacuum, $\beta=2\pi/N-\alpha$, h is the height of anode block and $D_1 = r_1 - r_a$.

Equivalent inductance and capacitance of large cavity are

$$L_2 = \frac{\mu_0 \alpha D_2}{6h}\left(2r_2 + r_a\right), \tag{5}$$

and

$$C_2 = \frac{2\varepsilon_0 h}{D_2 \alpha}\left(r_2 \ln\frac{r_2}{r_a} - D_2\right) + \frac{\varepsilon_0 h \beta}{\left(\pi\text{-}\alpha\text{-}\beta\right)\tan\left(\beta/2 + \alpha/2\right)}, \tag{6}$$

where $D_2 = r_2 - r_a$.

C_k is usually much smaller than C_1 and C_2, so the latter part of Eq. (1) can be neglected. Then a simplified expression of π-mode frequency is [1]

$$\omega_\pi = \sqrt{\frac{L_1 + L_2}{L_1 L_2 \left(C_1 + C_2\right)}}. \tag{7}$$

3. Results

CST-MWS, a finite-integration code for simulating electromagnetic fields, is adopted to build a rising-sun magnetron resonance system model and simulate π-mode resonance frequency. Π-mode electric field distribution is shown in Figure 3.

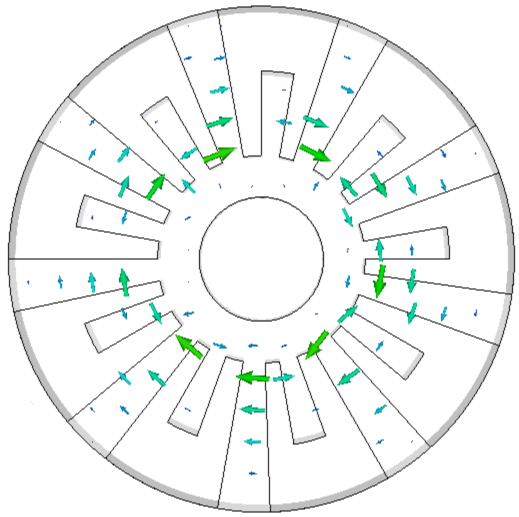

Fig. 3 Π-mode electric field distribution of rising-sun magnetron.

Two rising-sun magnetrons are taken as examples to verify the validity of the equivalent circuit method.

Structure dimensions of the first magnetron are r_c=1.50 *mm*, r_a=2.50 *mm*, r_1=7.00 *mm*, r_2=5.00 *mm*, *h*=10.00 *mm*, α=12°, while those of the second magnetron are r_c=1.22 *mm*, r_a=2.03 *mm*, r_1=4.95 *mm*, r_2=3.68 *mm*, *h*=4.83 *mm*, α=12°. The simulation and calculation results are shown in Table 1.

Tab. 1 Simulation and caculation results of two rising-sun magnetrons.

Magnetron NO.	1	2
Simulation (*GHz*)	16.59	25.61
Calculation (*GHz*)	17.23	26.63
Relative Error (%)	3.86	3.98

As can be seen from the Table 1, the equivalent circuit calculation results are consistent well with the simulation results, and the error is relatively small. So this method can be used to calculate the π-mode resonant frequency of rising-sun magnetron efficiently and accurately, and then advance the pre-design and analysis of the rising sun magnetron.

4. Conclusions

Equivalent circuit of rising-sun magnetron was analyzed in this paper. And π-mode frequency was calculated by expressions of equivalent capacitance and inductance of sector cavity. The formula of resonant frequency was simplified by analyzing the resonant characteristic of both big and small cavities. And then an expression of resonant frequency as a function of structure dimensions was derived. In addition, CST electromagnetic simulation software was adopted to create a rising-sun magnetron model. Simulation results of resonant frequencies were consistent well with theoretical calculation results, which verified the accuracy of equivalent circuit theory.

Acknowledgements

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